

الف) $y = x^3 - 3x^2 + 3x - 1 + 1 \rightarrow y = (x-1)^3 + 1 \sim R_y = \mathbb{R}$

$R_y = \mathbb{R} - (-1, 1]$

ب) $y = \frac{1}{x^2 - 2x} \rightarrow yx^2 - 2yx + 1 = 0 \rightarrow \Delta = 4y^2 - 4y(-1) = 4y^2 + 4y$
 $\sim \omega \rightarrow \frac{2 \pm \sqrt{4y^2 + 4y}}{2y} \rightarrow y \neq 0 \sim 4y^2 + 4y \geq 0 \rightarrow 4y(y+1) \geq 0 \rightarrow y \neq -1$

$\frac{-1 \pm \sqrt{1-1}}{-1 \pm 1}$

الف) $y = x^2 - 4x + 10 \rightarrow x_s = \frac{-(-4)}{2} = 2 \sim y_s = 4 - 8 + 10 = 6$

$\sim R_y = [6, +\infty)$

ب) $y = x^2 + 4x + 3 \rightarrow x_s = -2 \sim y_s = 4 - 8 + 3 = -1 \rightarrow (-\infty, -1]$

ج) $y = \sqrt{x^2 - 4x + 3} \rightarrow \Delta = 16 - 4(1)(-3) = 28 \sim \frac{-\Delta}{2a} = \frac{-28}{2} = -14 \sim y_s \rightarrow [-14, +\infty)$

د) $y = \sqrt{4x - x^2} \xrightarrow{a < 0} x_{max} = \frac{-4}{-2} = 2 \sim y_{max} = 2 \sim R_y = (-\infty, 2]$

الف) $R_y = \mathbb{R}$

ب) $R_y = \mathbb{R}$

ج) $R_y = [0, +\infty)$

د)

الف) $y = \frac{3x+1}{x-2} \rightarrow \mathbb{R} - \{3\} = R_y$

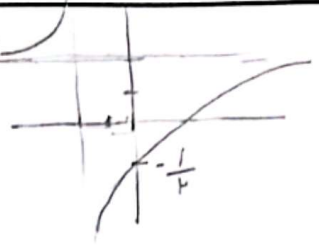
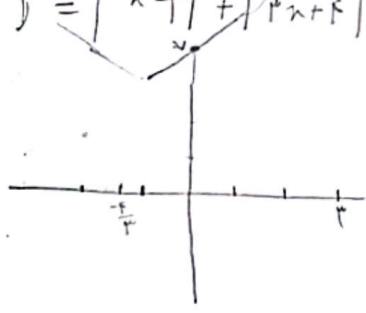
ب) $y = \frac{2x+1}{x+1} \sim \mathbb{R} - \{2\} = R_y$

الف) $y = \sqrt{\frac{4x+8}{x+1}} \rightarrow y^2x + y^2 = 4x + 8 \rightarrow y^2x - 4x = y^2 + 8 \rightarrow x(y^2 - 4) = y^2 + 8$

$\sim x = \frac{y^2 + 8}{y^2 - 4} \rightarrow y \neq 2, -2 \rightarrow \frac{-2}{+} \mid \frac{+2}{-} \mid \frac{+}{+} \sim R_y = \mathbb{R} - [-2, 2]$

ب) $y = \sqrt{\frac{4x+1}{x-2}} \rightarrow 4y^2 - y^2x = 4x + 1 \sim 4y^2 - 1 = 4x + y^2x$

$4y^2 - 1 = x(4 + y^2) \rightarrow x = \frac{4y^2 - 1}{4 + y^2} \sim \frac{-\frac{1}{2}}{+} \mid \frac{+\frac{1}{2}}{-} \mid \frac{+}{+} \sim R_y = \mathbb{R} - [\frac{1}{2}, \frac{1}{2}]$

ا) $y = \frac{x^2-1}{x+1}$ $-1 \sim \text{بؤلگه}$ $2 \sim \text{بؤلگه}$ 	ب) $y = \frac{x^2-1}{x+2}$ $-2 \sim \text{بؤلگه}$ $2 \sim \text{بؤلگه}$ 	6
ا) $y = \sin x + \frac{1}{\sin x} \rightarrow R_y = (-\infty, -1] \cup [1, +\infty)$ ب) $y = \frac{x^4+1}{x^4} = x^4 + \frac{1}{x^4} \rightarrow R_y = (-\infty, -1] \cup [1, +\infty)$ ج) $y = \frac{\sqrt{x^2+1}}{\sqrt{x}} \rightarrow \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow R_y = (-\infty, -1] \cup [1, +\infty)$ د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow R_y = [1, +\infty)$		7
ا) $y = x^2 + \frac{1}{x^2+1} \rightarrow x^2 + \frac{1}{x^2} + \frac{1}{x^2} \rightarrow R_y = [\frac{1}{3}, +\infty)$ ب) $y = \frac{x^2+5+1}{\sqrt{x^2+5}} = \sqrt{x^2+5} + \frac{1}{\sqrt{x^2+5}} \rightarrow$ $\sqrt{5} + \frac{1}{\sqrt{5}} = \sin = -1 \leftarrow \min$ $= \frac{5+1}{\sqrt{5}} = \frac{6\sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{6\sqrt{5}}{5} = \frac{6}{\sqrt{5}} = \frac{6\sqrt{5}}{5} = 2, \dots$ $R_y = [1, 0, +\infty)$		8
ا) $y = \left \frac{x^2-1}{x} \right + \left \frac{x^2+1}{x} \right \rightarrow$  $\begin{cases} x^2-1+x^2+1 & x > 0 \\ x^2-1-x^2-1 & -\frac{1}{x} < x < \frac{1}{x} \\ -x^2-1-x^2-1 & x < -\frac{1}{x} \end{cases}$ $\frac{x^2-1+x^2+1}{x} = \frac{2x^2}{x} = 2x$ $\frac{x^2-1-x^2-1}{x} = \frac{-2}{x}$ $\frac{-x^2-1-x^2-1}{x} = \frac{-2x^2-2}{x} = -2x - \frac{2}{x}$		9
ا) $y = \left \frac{x-1}{x} \right + \left \frac{x+1}{x} \right$ $\begin{matrix} & 1 & 2 & 3 \\ & 0 & 1 & 2 \\ & -1 & 0 & 1 \end{matrix}$ $R_y = [1, +\infty)$  ب) $y = \left \frac{x-1}{x} \right - \left \frac{x+1}{x} \right$ $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 0 & 0 \end{bmatrix}$ 		10