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حسابان، تمرین ۲، A۰۰

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الف) $y = x^3 - 3x^2 + 3x - 1 + 1 \to (x-1)^3 + 1 \to y - 1 = (x-1)^3$

$x = \sqrt[3]{y-1} + 1 \to R_f = \mathbb{R}$

ب) $y = \dfrac{1}{x^2 - 2x} \to yx^2 - 2yx - 1 = 0 \Rightarrow x = \dfrac{2y \pm \sqrt{4y^2 + 4y}}{2y}$

$\to 4y^2 + 4y \geq 0 \to y^2 + y \geq 0 \Rightarrow y(y+1)$ $\quad$ (-1, 0: $+\ -\ +$)

$R_f = (-\infty, -1] \cup [0, +\infty)$

۲- الف) $y = x^2 - 4x + 10$ $\quad$ min $\left| \begin{array}{l} 4/2 = 2 \\ 4 - 8 + 10 = 6 \end{array} \right.$ $\quad R_f = [6, +\infty)$

ب) $y = -x^2 + 6x + 3$ $\quad$ max $\left| \begin{array}{l} -6/-2 = 3 \\ -9 + 18 + 3 = 12 \end{array} \right.$ $\quad R_f = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 3}$ $\quad$ min $\left| \begin{array}{l} 4/2 = 2 \\ 4 - 8 - 3 = -7 \end{array} \right.$ $\to \sqrt{[-7, +\infty)} = [0, +\infty)$

$R_f = \nearrow$

د) $\sqrt{-x^2 + 6x} \to$ max $\left| \begin{array}{l} -6/-2 = 3 \\ -9 + 18 = 9 \end{array} \right.$ $\to \sqrt{(-\infty, 9]} = [0, 3]$

$R_f = \nearrow$

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۳) الف) $x^3 - 5x^2 + 3x + 1 \to R_f = \mathbb{R}$

ب) $x^7 - 6x^4 + 3x + 2 \to R_f = \mathbb{R}$

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ج) $\sqrt{x^4 - 4x^2 + 4x + 1} \rightarrow \sqrt{\mathbb{R}} = [0, +\infty) = R_f$

د) $y = (x^3 - 4x^2 + 2x + 1)^5 \rightarrow R_f = [0, +\infty)$

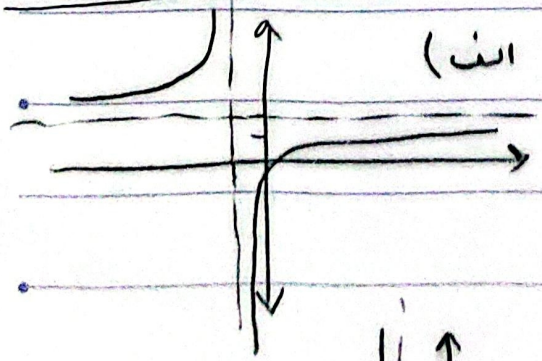
٤- أ) $y = \frac{2x+1}{x-2} \rightarrow R_f = \mathbb{R} - \{2\}$

ب) $y = \frac{4x+1}{x+1} \rightarrow R_f = \mathbb{R} - \{4\}$

٥- أ) $y = \sqrt{\frac{2x+4}{x+1}} \rightarrow \sqrt{\mathbb{R} - \{1\}} \rightarrow R_f = [0, +\infty) - \{\sqrt{1}\}$

ب) $y = \sqrt{\frac{2x+1}{-x+2}} \rightarrow \sqrt{\mathbb{R} - \{-2\}} \rightarrow R_f = [0, +\infty)$

٦- أ)

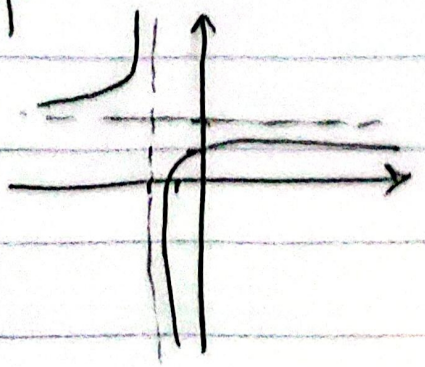


$R_f = \mathbb{R} - \{2\}$

خط مقارب $x = -1$

نقطة $= 2$

ب)



$R_f = \mathbb{R} - \{2\}$

خط مقارب $x = 2$

نقطة $x = 2$

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الف) $y = \sin + \frac{1}{\sin} \xrightarrow{1 \geq \sin \geq -1} R_f = (-\infty, -2] \cup [2, +\infty)$

ب) $y = \frac{x^4+1}{x^3} = x^3 + \frac{1}{x^3} \to R_f = (-\infty, -2] \cup [2, +\infty)$

ج) $y = \frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}} = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} \to R_f = (-\infty, -2] \cup [2, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \to R_f = [2, +\infty)$

-۸ الف) $x^2 + \frac{1}{x^2+3} + 3 - 3 \to \left(x^2+3+\frac{1}{x^2+3}\right) - 3 \to \frac{10}{3} - 3 = \frac{1}{3}$

کمترین $= 10/3$ $\Rightarrow R_f = [\frac{1}{3}, +\infty)$

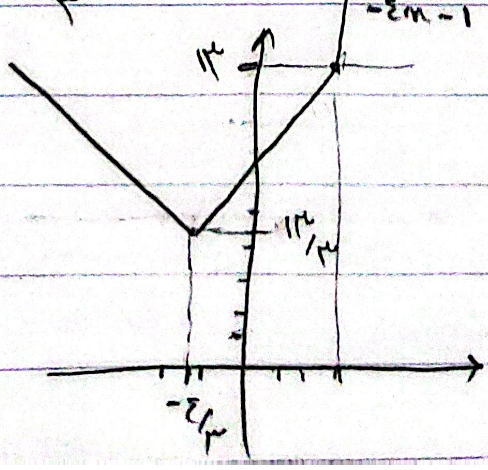
ب) $y = \frac{x^2+5}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \to R_f = [5/2, +\infty)$

کمترین $= 5/2$

-۹ $y = |x-3| + |3x+4| =$

3 ، $-4/3$

$-4/3$		3	
$-x+3-3x-4 = -4x-1$	$-x+3+3x+4 = 2x+7$	$x-3+3x+4 = 4x+1$	



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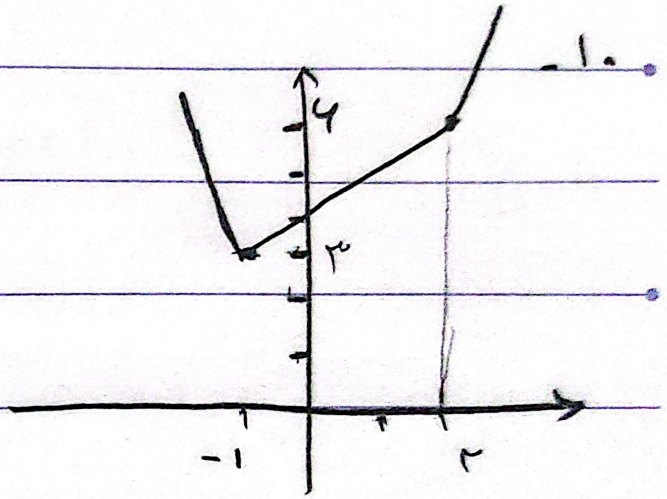
ب۱۱) $y = |x-2| + |2x+2|$

$x=2 \leftarrow$ $\quad x=-1 \leftarrow$

$x<-1$	$-1 \le x < 2$	$x \ge 2$
$-(x-2)-(2x+2)$	$-(x-2)+(2x+2)$	$(x-2)+(2x+2)$
$= -3x$	$= x+4$	$= 3x$



ب۱۲) $y = |2x-2| - |x+1|$

$x=1 \leftarrow$ $\quad x=-1 \leftarrow$

$x<-1$	$-1 \le x < 1$	$x \ge 1$
$-(2x-2)+(x+1)$	$2x-2-x-1$	$(2x-2)-(x+1)$
$= -x+3$	$= x-3$	$= x-3$