

نکته: یاد هم داشته باشید

نام: خانم آریس بهار مشهدیان

الف) $y = x^3 - 3x^2 + 3x + 1 - 1$

$y = (x-1)^3 + 1$

$y-1 = (x-1)^3$

$(y-1)^3 = x-1$

$x = (y-1)^3 + 1 \rightarrow R = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x + 1}$

$\textcircled{1} x^2 - 2x \neq 0 \rightarrow x(x-2) \neq 0 \rightarrow x \neq 0$
 $x \neq 2$

$y x^2 - 2yx = 1 \rightarrow y x^2 - 2yx - 1 = 0$

$\Delta = 4y^2 - 4 \times y \times -1 = 4y^2 + 4y > 0 \rightarrow \frac{-1 \pm \sqrt{1+4y}}{2}$
 $4y(y+1) > 0$

$R = (-\infty, -1) \cup (0, \infty)$ ← از این عبارت

الف) $y = x^2 - 4x + 10$

$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \frac{4 \pm \sqrt{16 - 40}}{2} = 2 \pm \sqrt{-4}$

$R = [4, \infty)$

ب) $y = -x^2 + 4x + 3$

$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 + 12}}{2} = 2 \pm \sqrt{13}$

$R = (-\infty, 13]$

ج) $y = \sqrt{x^2 - 4x - 3}$

$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 + 12}}{2} = 2 \pm \sqrt{13}$

$\sqrt{E \cup \infty}$

$R = [0, \infty)$

د) $y = \sqrt{4x - x^2}$

$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4 - 4}}{2} = 1$

$\sqrt{(-\infty, 4]}$

$R = [0, 2]$

الف) $y = x^3 - 5x^2 + 2x + 1$

$R = \mathbb{R}$

ب) $y = x^3 - 4x^2 + 3x + 2$

$R = \mathbb{R}$

ج) $y = \sqrt{x^2 - 4x^2 + 5x + 1}$

$x^2 - 4x^2 + 5x + 1$
 $R = \mathbb{R}$
 $\sqrt{R} = [0, \infty)$

$R = [0, \infty)$

د) $y = (x^2 - 4x^2 + 5x + 1)^2$

$x^2 - 4x^2 + 5x + 1$
 $R = \mathbb{R}$
 $Ry = [0, \infty)$

ب) $y = \frac{2x+1}{x+1}$

$ad \neq cb, c \neq 0$
 $2 \neq 1, 1 \neq 0 \checkmark$

$R = \mathbb{R} - \{1\}$

الف) $y = \frac{2x+1}{x-1}$

$ad \neq cb, c \neq 0$
 $2 \neq 1, 1 \neq 0 \checkmark$

$R = \mathbb{R} - \{1\}$

الف) $y = \sqrt{\frac{2x+1}{x+1}}$

$y = \frac{2x+1}{x+1} \rightarrow yx + y = 2x + 1$

$\rightarrow yx - 2x = 1 - y \rightarrow x(y-2) = 1-y \rightarrow x = \frac{1-y}{y-2}$

$\mathbb{R} - \{2\}$

$\leftarrow y \neq 2$

$\rightarrow y = \sqrt{\mathbb{R} - \{2\}} \Rightarrow R = [0, \infty) - \{\sqrt{2}\}$

ب) $y = \sqrt{\frac{2x+1}{2-x}}$

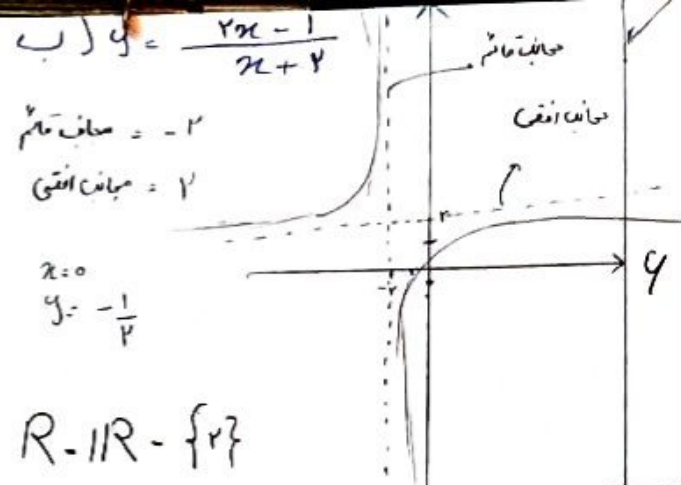
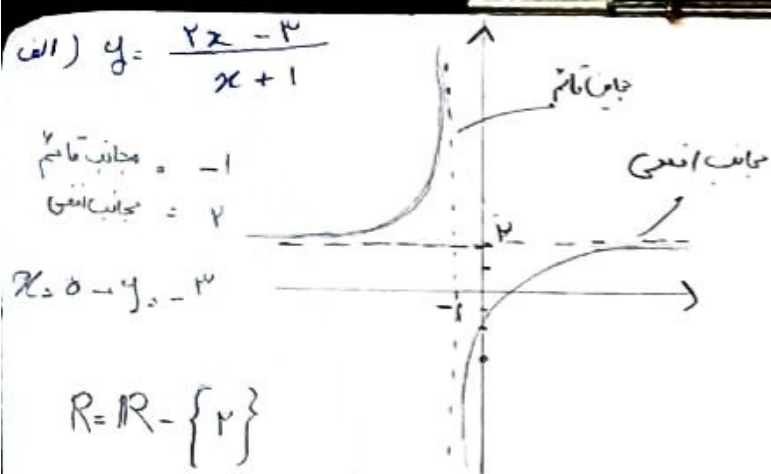
$y = \frac{2x+1}{2-x} \rightarrow x = \frac{-2y+1}{-y-2}$

$R = \mathbb{R} - \{2\} \leftarrow y \neq 2$

$\sqrt{R - \{2\}} = [0, \infty) - \{\sqrt{2}\}$

$R = [0, \infty)$

Check: $\frac{2x+1}{2-x} = 2$
 $2x+1 = 4-2x$
 $4x = 3$
 $x = \frac{3}{4}$



الف) $\sin x + \frac{1}{\sin x}$

$a + \frac{1}{a}$

$-1 \leq \sin x \leq 1$

$-1 + (-\frac{1}{-1}) = -2$

$R = (-\infty, -2] \cup [2, \infty)$

ب) $\frac{x^3+1}{x^3}$

$x^3 + \frac{1}{x^3}$

$R = (-\infty, -1] \cup [1, \infty)$

ج) $\frac{\sqrt[3]{x^3+1}}{\sqrt[3]{x}}$

$\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$

$R = (-\infty, -1] \cup [1, \infty)$

د) $\sqrt{x} + \frac{1}{\sqrt{x}}$

$R = [2, \infty)$

الف) $x^3 + \frac{1}{x^3+3}$

$x^3+3-3 + \frac{1}{x^3+3}$

$\rightarrow (x^3+3) + \frac{1}{(x^3+3)} - 3$

$R = [\frac{1}{3}, \infty)$

$\lim_{x \rightarrow \infty} \frac{1}{x^3+3} = 0 \rightarrow 3 + \frac{1}{3} - 3 = \frac{1}{3}$

ب) $\frac{x^3+a}{\sqrt{x^3+5}}$

$\frac{x^3+5+1}{\sqrt{x^3+5}} = \sqrt{x^3+5} + \frac{1}{\sqrt{x^3+5}}$

$x=0 \rightarrow \sqrt{5} + \frac{1}{\sqrt{5}}$

$R = [\frac{6}{5}, \infty)$

$y = |x-3| + |3x+5|$

$\rightarrow \textcircled{1}$

$\rightarrow \textcircled{2}$

$x-3+3x+5$

$x > 3$

$-x+3+3x+5$

$\frac{5}{3} \leq x \leq 3$

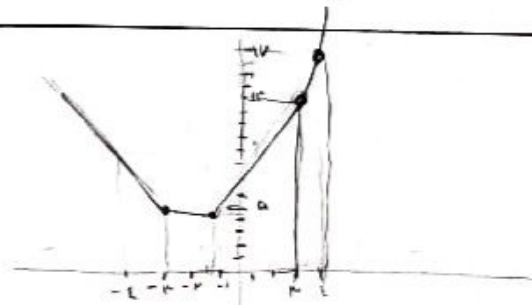
$-x+3-3x-5$

$x < -\frac{5}{3}$

$\rightarrow \begin{cases} 4x+2 & x > 3 \\ 2x+8 & \frac{5}{3} \leq x \leq 3 \\ -4x-2 & x < -\frac{5}{3} \end{cases}$

$\frac{5}{3} = -\frac{5}{3} + \frac{10}{3} = \frac{10}{3}$

$R = [\frac{10}{3}, \infty)$



الف) $y = |x-3| + |3x+5|$

$\rightarrow \textcircled{1}$

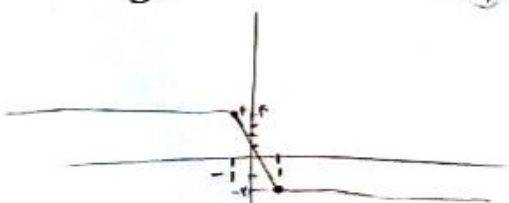
$\rightarrow \textcircled{2}$

$[3, \infty)$

ب) $y = |3x-2| - |x+1|$

$\rightarrow \textcircled{1}$

$\rightarrow \textcircled{2}$



$[-1, \frac{2}{3}]$