$y = x^3 - 3x^2 + 3x = (x-1)^3 + 1 \to y = (x-1)^3 - 1$

۱۹

نگار صدران

۱ الف

$(x-1)^3 = y - 1 \to x = \sqrt[3]{y-1} + 1$

$R_f \ (+\infty, -\infty)$

---

۱ ب

$y = \dfrac{1}{x^2 - 2x} \Rightarrow yx^2 - 2xy - 1 = 0$

۵

$x_{1,2} = \dfrac{+2y \pm \sqrt{4y^2 + 4y}}{2y}$

$y \neq 0 \ , \ 4y^2 + 4y \geq 0 \to$

y		-1		0	
$4y^2+4y$	$+$		$-$		$+$

$R_f \ [-\infty, -1], (0, +\infty)$

---

۲ الف

$y = x^2 - 4x + 1 \ , \to -\dfrac{b}{2a} = \dfrac{4}{2} = 2 \to y_{min} = 6$

$R_f \ [6, +\infty)$

---

۲ ب

$y = -x^2 + 6x + 3 \to -\dfrac{b}{2a} = 3 \to y_{max} = 12$

$R_f \ [-\infty, +12]$

۵

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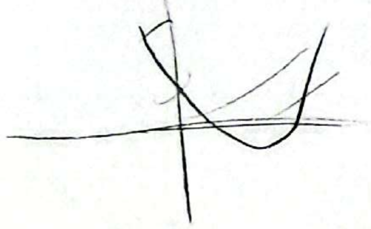
۳ الف

$y = \sqrt{x^2 - 4x - 3} \to -\dfrac{b}{2a} = 2 \to y_{min} = -7$

$R_f \ [-7, +\infty)$

$x^2 - 4x - 3$



$R_f \ [0, +\infty]$

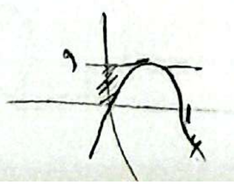
$\sqrt{x^2 - 4x}$

---

۳ ب

$y = \sqrt{6x - x^2} \to y = -x^2 + 6x \to -\dfrac{b}{2a} = 3 \to y_{max} = 9$

$R_f = [0, 3]$



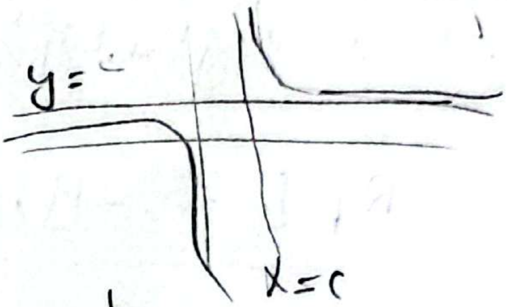
(۳ الف) $y = x^4 - 5x^3 + 2x + 1 \rightarrow R_f = R$

(۳ ب) $y = x^7 - 6x^4 + 2x + 2 \rightarrow R_f = R$

(۳ ج) $y = \sqrt{x^7 - 6x^3 + 5x - 1} \rightarrow R_f\ [0, +\infty)$

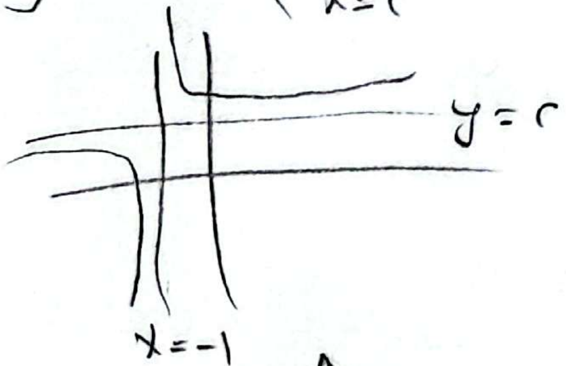
(۳ د) $y = (x^5 - 4x^1 + 2x + 1)^2 \rightarrow R_f\ [0, +\infty)$

(۴ الف) $y = \dfrac{2x+1}{x-3}$



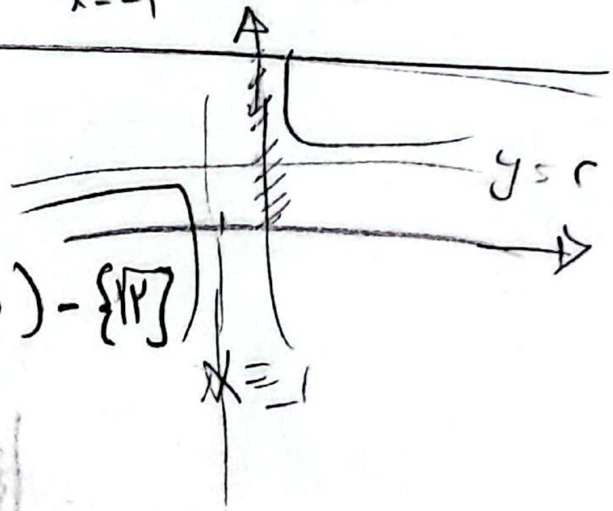
$R_f = R - \{3\}$

(۴ ب) $y = \dfrac{2x+1}{x+1}$



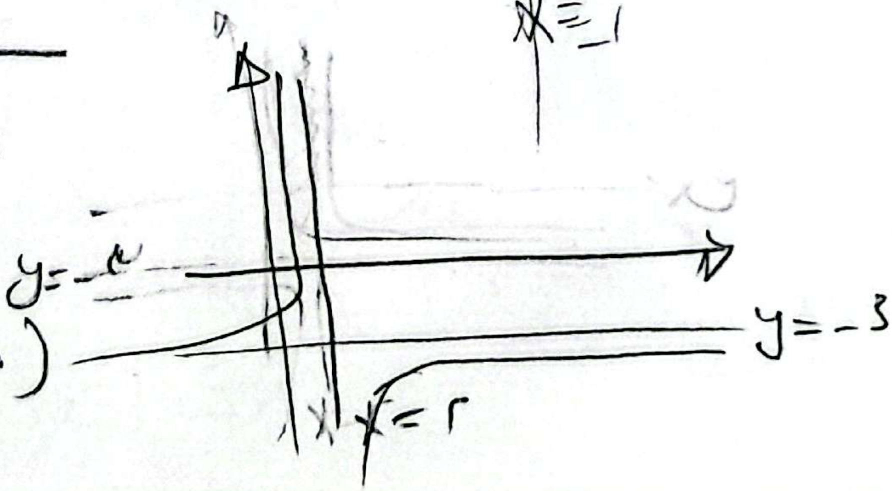
$R_f = R - \{2\}$

$y = \sqrt{\dfrac{2x+4}{x+1}} \rightarrow$



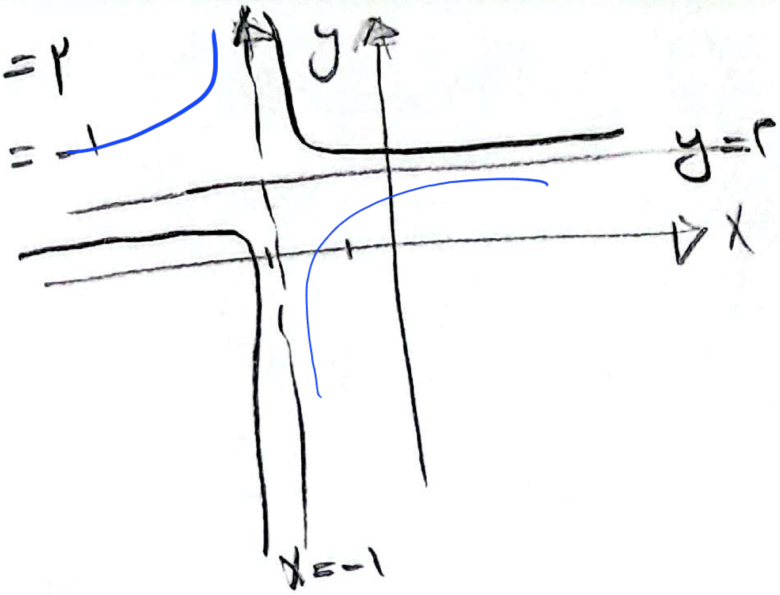
$R_f = [0, +\infty) - \{\sqrt{2}\}$

$y = \sqrt{\dfrac{2x+1}{-x+2}}$



$R_f\ [0, +\infty)$

۶ الف $y=\frac{۲x-۴}{x+۱} \longrightarrow \begin{matrix} y=۲ \\ x=-۱ \end{matrix}$



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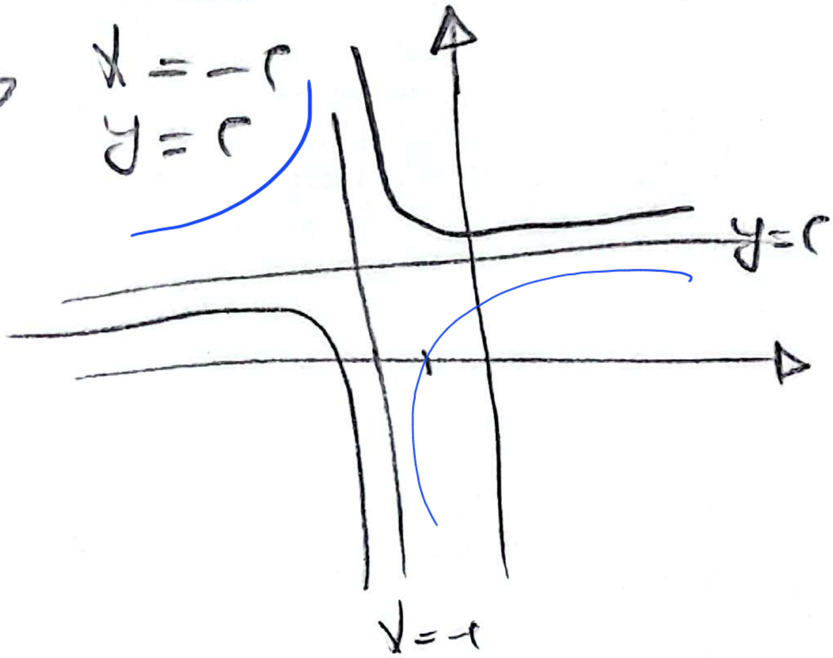
۶ ب $y=\frac{۲x-۱}{x+۲} \longrightarrow \begin{matrix} x=-۲ \\ y=۲ \end{matrix}$



۷ الف $y=\sin x+\frac{۱}{\sin x}$ $R_f=(-\infty, -۲] \cup [۲, +\infty)$

۵

۷ ب $y=\frac{x^۴+۱}{x^۲}=x^۲+\frac{۱}{x^۲}$

$R_f=[-\infty, -۲] \cup [۲, +\infty)$

۷ ج $y=\frac{\sqrt[۳]{x^۲}+۱}{\sqrt[۳]{x}}=\sqrt[۳]{x}+\frac{۱}{\sqrt[۳]{x}}$ $R_f=[-\infty, -۲] \cup [۲, +\infty)$

$\frac{7}{2}$ $y=\sqrt{x}+\frac{1}{\sqrt{x}} \rightarrow R_f=[2,+\infty)$

$\frac{8}{\text{ند}}$ $y=x^2+\frac{1}{x^2+3} \rightarrow y=(x^2+3)+\frac{1}{x^2+3}-3$

$y=a+\frac{1}{a}-3$

$a>0 \longrightarrow R_f=[\frac{1}{[illegible]},+\infty)$

$\frac{8}{3}$ $y=\frac{x^2+5}{\sqrt{x^2+4}}=\frac{\sqrt{x^2+4}\times\sqrt{x^2+4}+1}{\sqrt{x^2+4}}$

$y=\sqrt{x^2+4}+\frac{1}{\sqrt{x^2+4}}+1$

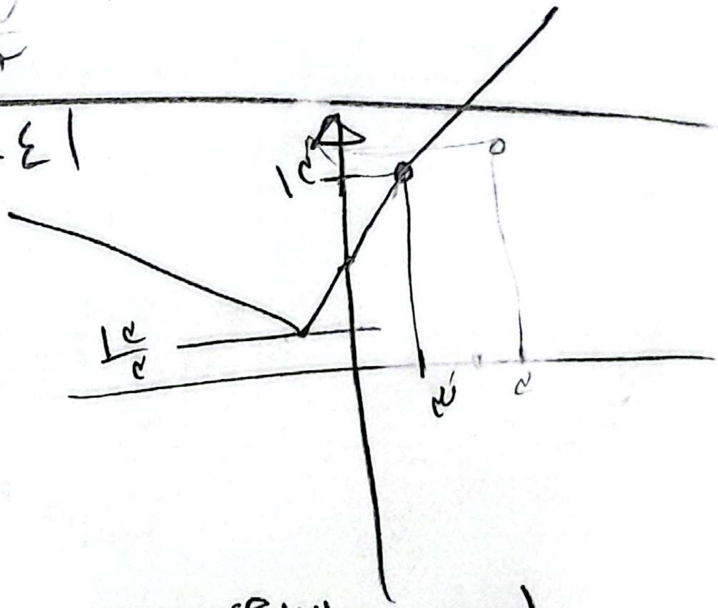
$\min\sqrt{x^2+4}=2 \longrightarrow R_f=[\frac{5}{2},+\infty)$

$2+\frac{1}{2}\neq\frac{5}{2}$ $2+\frac{1}{2}=\frac{5}{2}$

---

9 $y=|x-3|+|3x+4|$

$-\frac{4}{3}$		3
$-3x-4$	$3x+4$	$3x+4$
$-x+3$	$-x+3$	$x-3$
$-4x-1$	$2x+7$	$4x+1$



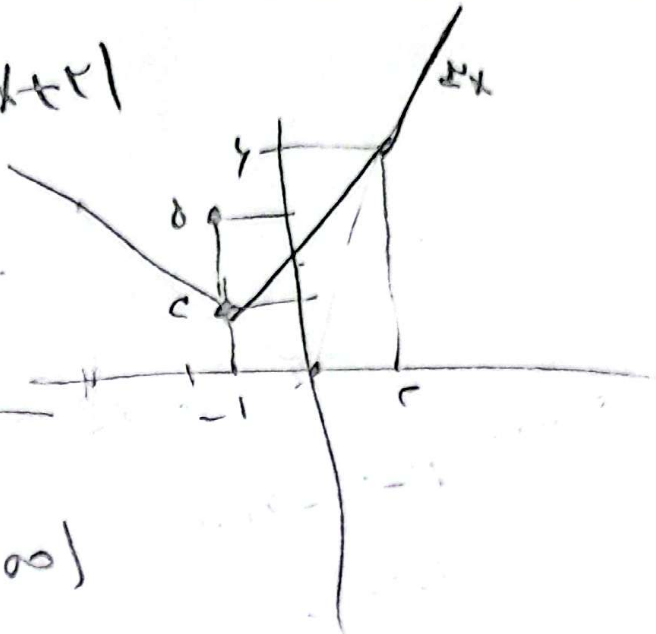
$R_f=[\frac{13}{3},+\infty)$

$$f(x)=\begin{cases}4x+1 & x\geq 3\\ 2x+7 & -\frac{4}{3}\leq x<3\\ -4x-1 & x\leq-\frac{4}{3}\end{cases}$$

١٥

الف) $y = |x-٢| + |٢x+٢|$

	-١	٢
$-x+٢$	$-x+٢$	$x-٢$
$-٢x-٢$	$٢x+٢$	$٢x+٢$
$-٣x$	$x+٤$	$٣x$



$R_f = [٣, +\infty)$

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١٠ ب

$y = |٢x-٢| - |x+١|$

	-١	١
$+x+١$	$-x-١$	$-x-١$
$-٢x+٢$	$-٢x+٢$	$٢x-٢$
$-x+٣$	$-٣x+١$	$x-٣$

$y=٤$ $\quad$ $y=٢$

$R_f = [-٢, +\infty)$