نگار صران

$y = x^3 - 3x^2 + 3x = (x-1)^3 + 1 \rightarrow y = (x-1)^3 - 1$

۱ الف

$(x-1)^3 = y - 1 \rightarrow x = \sqrt[3]{y-1} + 1$

$R_f\,(+\infty, -\infty)$

---

۱ ب

$y = \dfrac{1}{x^2 - 2x} \Rightarrow yx^2 - 2xy - 1 = 0$

$x_{1,2} = \dfrac{+2y \pm \sqrt{4y^2 + 4y}}{2y}$

$y \neq 0 \;,\; 4y^2 + 4y \geq 0 \longrightarrow$

| $y$ | | $-1$ | | $0$ | |
|---|---|---|---|---|---|
| $4y^2+4y$ | $+$ | | $-$ | | $+$ |

$R_f\,[-\infty, -1], (0, +\infty)$

---

۲ الف

$y = x^2 - 4x + 10 \rightarrow \dfrac{-b}{2a} = \dfrac{4}{2} = 2 \rightarrow y_{min} = 6$

$R_f\,[6, +\infty)$

---

۲ ب

$y = -x^2 + 6x + 3 \rightarrow \dfrac{-b}{2a} = 3 \rightarrow y_{max} = 12$

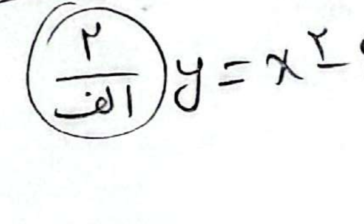
$R_f\,[-\infty, +12]$

---

۳ ج

$y = \sqrt{x^2 - 4x - 3} \rightarrow -\dfrac{b}{2a} = 2 \rightarrow y_{min} = -7$

$x^2 - 4x - 3 \quad R_f\,[-7, +\infty)$



$R_f\,[0, +\infty]$

$\sqrt{x^2 - 4x}$

---

۳ د

$y = \sqrt{\frac{6}{x} x^2} \rightarrow y = -x^2 + 6x \rightarrow \dfrac{-b}{2a} = 3 \rightarrow y_{max} = 9$

$R_f = [0, 3]$

(۲ الف) $y = x^4 - 5x^3 + 2x + 1 \longrightarrow Rf = R$

(۲ ب) $y = x^7 - 6x^4 + 5x + 2 \longrightarrow Rf = R$

(۳ ج) $y = \sqrt{x^7 - 6x^3 + 5x - 1} \longrightarrow Rf\ [0, +\infty)$

(۳ د) $y = (x^5 - 4x^1 + 2x + 1)^2 \longrightarrow Rf\ [0, +\infty)$

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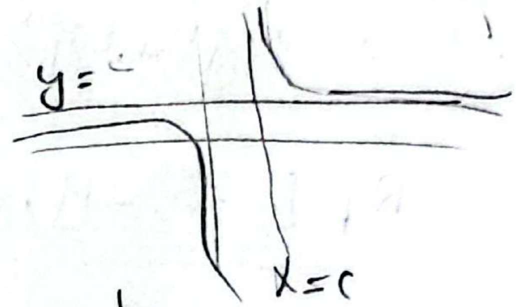
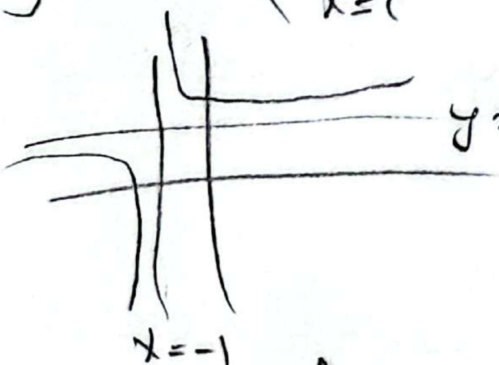
(۴ الف) $y = \dfrac{2x+1}{x-2}$




$Rf = R - \{2\}$

(۴ ب) $y = \dfrac{2x+1}{x+1}$




$Rf = R - \{2\}$

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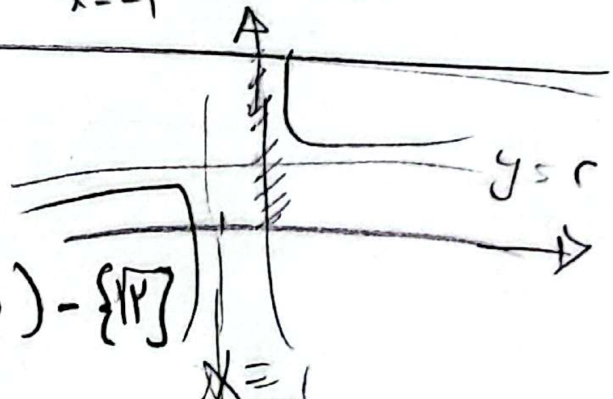
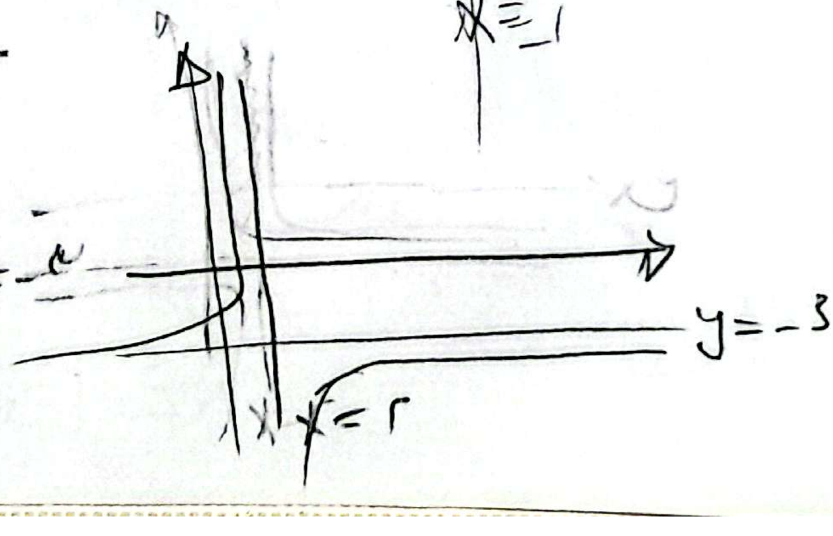
$y = \sqrt{\dfrac{2x+4}{x+1}} \longrightarrow$



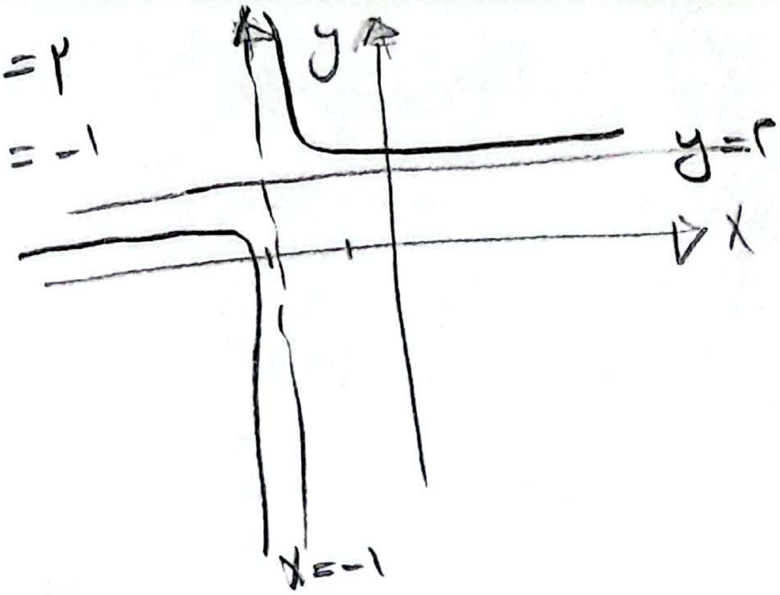

$Rf = [0, +\infty) - \{\sqrt{2}\}$

$y = \sqrt{\dfrac{2x+1}{-x+2}}$




$Rf\ [0, +\infty)$

$\frac{6}{\text{الف}}$ $y=\frac{2x-2}{x+1} \longrightarrow y=2$, $x=-1$

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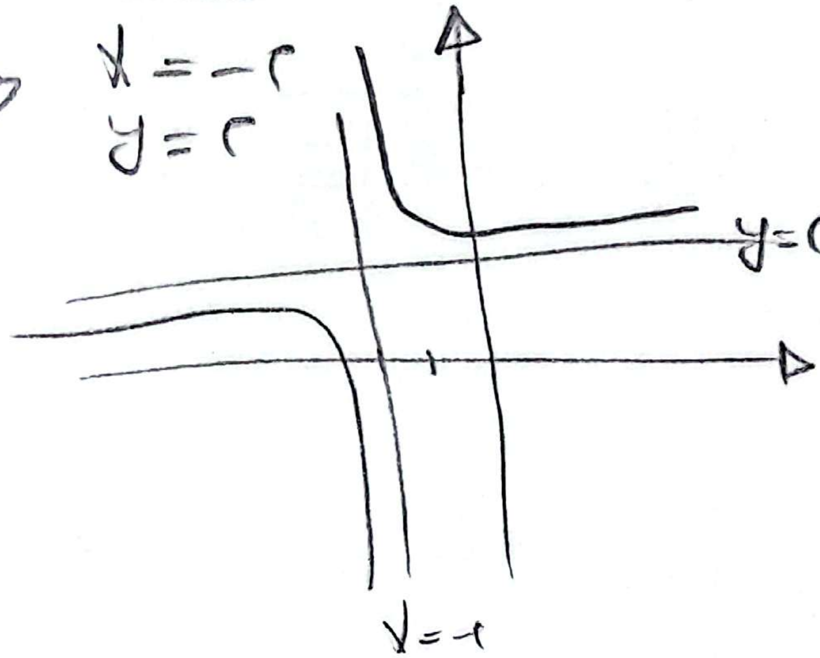
$\frac{6}{\text{ب}}$ $y=\frac{2x-1}{x+2} \longrightarrow x=-2$, $y=2$

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$\frac{7}{\text{الف}}$ $y=\sin x+\frac{1}{\sin x}$ $R_f=(-\infty,-2]\cup[2,+\infty)$

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$\frac{7}{\text{ب}}$ $y=\frac{x^4+1}{x^2}=x^2+\frac{1}{x^2}$

$R_f=[-\infty,-2]\cup[2,+\infty)$

---

$\frac{7}{2}$ $y=\frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}}=\sqrt[3]{x}+\frac{1}{\sqrt[3]{x}} \underset{R_f}{=} [-\infty,-2]\cup[2,+\infty)$

**٧ / د**

$$y=\sqrt{x}+\frac{1}{\sqrt{x}} \longrightarrow R_f=[2,+\infty)$$

---

**٨ / الف**

$$y=x^2+\frac{1}{x^2+3} \longrightarrow y=(x^2+3)+\frac{1}{x^2+3}-3$$

$$y=a+\frac{1}{a}-3$$

$$a\geq 3 \longrightarrow R_f=\left[\frac{1}{3},+\infty\right)$$

---

**٨ / ب**

$$y=\frac{x^2+5}{\sqrt{x^2+4}}=\frac{\sqrt{x^2+4}\times\sqrt{x^2+4}+1}{\sqrt{x^2+4}}$$

$$y=\sqrt{x^2+4}+\frac{1}{\sqrt{x^2+4}}$$

$$\min\sqrt{x^2+4}=2 \longrightarrow R_f=\left[\frac{5}{2},+\infty\right)$$

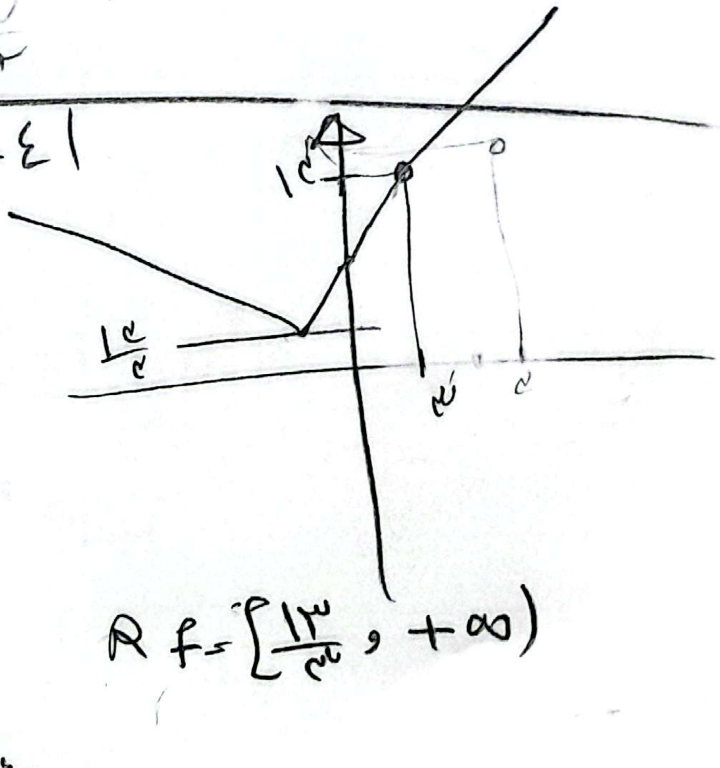
$$2+\frac{1}{2}=\frac{5}{2}$$

---

**٩** $y=|x-3|+|3x+4|$

| $-\frac{4}{3}$ | | $3$ |
|---|---|---|
| $-3x-4$ | $3x+4$ | $3x+4$ |
| $-x+3$ | $-x+3$ | $x-3$ |
| $-4x-1$ | $2x+7$ | $4x+1$ |

$$f(x)=\begin{cases}4x+1 & x\geq 3\\ 2x+7 & -\frac{4}{3}\leq x<3\\ -4x-1 & x\leq-\frac{4}{3}\end{cases}$$

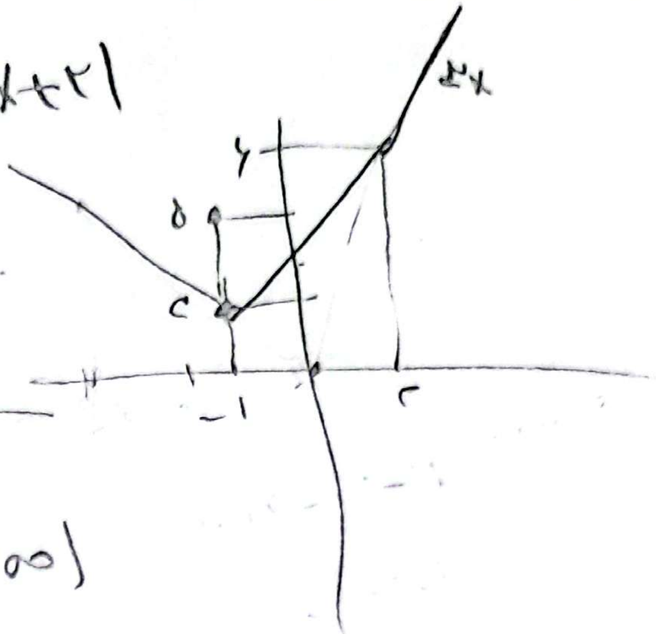


$$R_f=\left[\frac{13}{3},+\infty\right)$$

۱۵

الف $y=|x-2|+|2x+2|$

| | $-1$ | $2$ |
|---|---|---|
| $-x+2$ | $-x+2$ | $2x+2$ |
| $-2x-2$ | $2x+2$ | $x-2$ |
| $-3x$ | $x+4$ | $3x$ |




$R_f=[3,+\infty)$

۱۵ ب

$y=|2x-2|-|x+1|$

| | $-1$ | $1$ |
|---|---|---|
| $+x+1$ | $x+1$ | $-x-1$ |
| $-2x+2$ | $-2x+2$ | $2x-2$ |
| $-x+3$ | $-3x+1$ | $x-3$ |

$y=4$ $\quad$ $y=2$

$R_f=[-2,+\infty)$