۱) کرنیا دیباجی — یازدهم ریاضی A — ۲۰

الف) $y = x^3 - 3x^2 + 3x \Rightarrow y - 1 = (x-1)^3 \Rightarrow x - 1 = \sqrt[3]{y-1} \Rightarrow x = \sqrt[3]{y-1} + 1 \Rightarrow R_f = \mathbb{R}$

د

ب) $y = \dfrac{1}{x^2 - 2x} \Rightarrow yx^2 - 2xy - 1 = 0 \Rightarrow \Delta = 4y^2 - (4 \times -1 \times y) = 4y^2 + 4y$

$\Rightarrow x = \dfrac{-b \pm \sqrt{\Delta}}{2a} \Rightarrow \dfrac{+2y \pm \sqrt{4y^2 + 4y}}{2y} \Rightarrow \begin{cases} 4y^2 + 4y \geq 0 \Rightarrow 4y(y + 1) \geq 0 \Rightarrow \begin{cases} y = 0 \\ y = -1 \end{cases} \\ 2y \neq 0 \Rightarrow y \neq 0 \end{cases}$

$\Rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$

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۲)

الف) $y = x^2 - 4x + 10 \xrightarrow{a>0} \begin{cases} x_{min} = \dfrac{-b}{2a} = \dfrac{4}{2} = 2 \\ y_{min} = 4 - 8 + 10 = 6 \end{cases} \Rightarrow R_f = [6, +\infty)$

د

ب) $y = -x^2 + 6x + 3 \xrightarrow{a<0} \begin{cases} x_{max} = \dfrac{-b}{2a} = \dfrac{-6}{-2} = 3 \\ y_{max} = -9 + 18 + 3 = 12 \end{cases} \Rightarrow R_f = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 3} \xrightarrow{a>0} \begin{cases} x_{min} = \dfrac{-b}{2a} = \dfrac{4}{2} = 2 \\ y_{min} = 4 - 8 - 3 = -7 \end{cases} \Rightarrow R_f = \sqrt{[-7, +\infty)} = [0, +\infty)$

د) $y = \sqrt{6x - x^2} \xrightarrow{a<0} \begin{cases} x_{max} = \dfrac{-b}{2a} = \dfrac{-6}{-2} = 3 \\ y_{max} = -9 + 18 = 9 \end{cases} \Rightarrow R_f = \sqrt{(-\infty, 9]} \Rightarrow [0, 3]$

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۳)

الف) $y = x^3 - 5x^2 + 2x + 1 \Rightarrow R_f = \mathbb{R}$

ب) $y = x^7 - 6x^4 + 3x + 2 \Rightarrow R_f = \mathbb{R}$

ج) $\sqrt{x^7 - 6x^2 + 5x + 1} \Rightarrow R_f = [0, +\infty)$

د) $(x^5 - 4x^2 + 3x + 1)^2 \Rightarrow R_f = [0, +\infty)$

د

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۴)

الف) $y = \dfrac{3x+1}{x-2} \Rightarrow \begin{cases} c \neq 0 \\ 3 \times -2 \neq 1 \times 1 \end{cases} \Rightarrow R_f = \mathbb{R} - \{3\}$

د

ب) $y = \dfrac{2x+1}{x+1} \Rightarrow \begin{cases} c \neq 0 \\ 2 \times 1 \neq 1 \times 1 \end{cases} \Rightarrow R_f = \mathbb{R} - \{2\}$

۵

الف) $y=\sqrt{\frac{2x+4}{x+1}} \Rightarrow R_f=[0,+\infty)-\{\sqrt{2}\}$

ب) $\sqrt{\frac{3x+1}{2-x}} \Rightarrow \sqrt{\frac{3x+1}{-x+2}} \Rightarrow R_f=[0,+\infty)-\{\sqrt{3}\}$ (غلط ✗) $\Rightarrow R_f=[0,+\infty)$

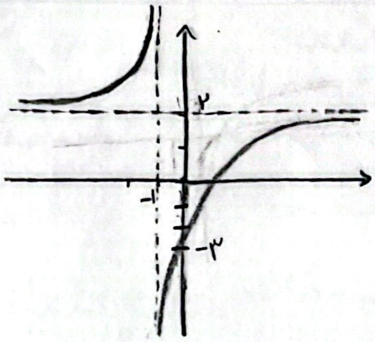
۶

الف) $y=\frac{2x-3}{x+1} \Rightarrow R_f=\mathbb{R}-\{2\}$

مجانب قائم $=-1$

مجانب افقی $=2$

محل برخورد با محور y ها $=\frac{-3}{1}=-3$

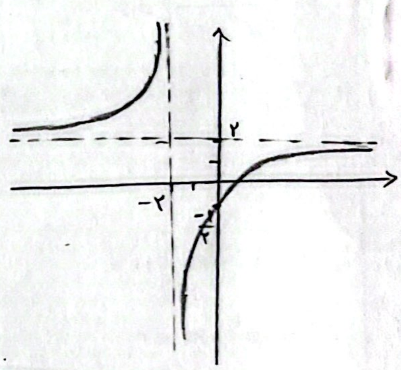


ب) $y=\frac{2x-1}{x+2} \Rightarrow R_f=\mathbb{R}-\{2\}$

مجانب قائم $=-2$

مجانب افقی $=2$

محل برخورد با محور y ها $=\frac{-1}{2}$



۷

الف) $y=\sin x+\frac{1}{\sin x} \Rightarrow R_f=(-\infty,-2]\cup[2,+\infty)$

ب) $y=\frac{x^6+1}{x^3} \Rightarrow \frac{x^6}{x^3}+\frac{1}{x^3} \Rightarrow x^3+\frac{1}{x^3} \Rightarrow R_f=(-\infty,-2]\cup[2,+\infty)$

ج) $\frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}} \Rightarrow \sqrt[3]{\frac{x^2}{x}}+\frac{1}{\sqrt[3]{x}} \Rightarrow \sqrt[3]{x}+\frac{1}{\sqrt[3]{x}} \Rightarrow R_f=(-\infty,-2]\cup[2,+\infty)$

د) $y=\sqrt{x}+\frac{1}{\sqrt{x}} \Rightarrow R_f=[2,+\infty)$

۸

الف) $y=x^2+\frac{1}{x^2+3} \to x^2+3=t \Rightarrow x^2=t-3 \Rightarrow t+\frac{1}{t}-3 \Rightarrow R_f=[\frac{1}{3},+\infty)$

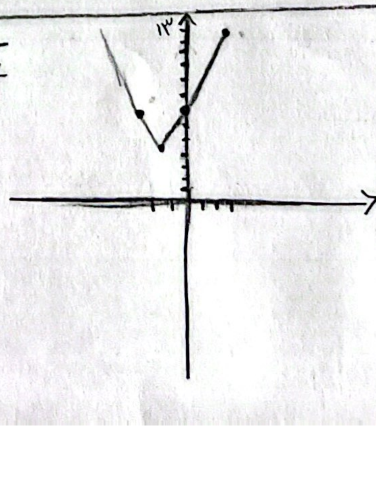
ب) $y=\frac{x^2+5}{\sqrt{x^2+4}} \Rightarrow \frac{x^2+4+1}{\sqrt{x^2+4}} \Rightarrow \sqrt{x^2+4}+\frac{1}{\sqrt{x^2+4}} \Rightarrow R_f=[\frac{5}{2},+\infty)$

۹

$y=|x-3|+|3x+4| \Rightarrow$

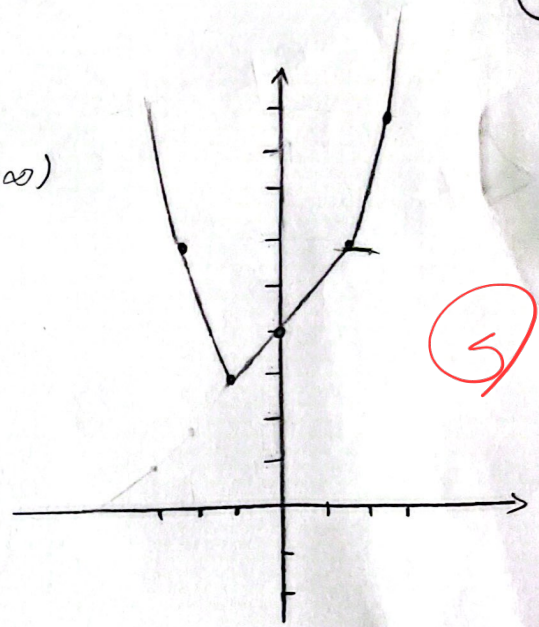
$x<-\frac{4}{3}$	$-\frac{4}{3}\le x<3$	$x\ge 3$
$-x+3-3x-4$	$-x+3+3x+4$	$x-3+3x+4$
$-4x-1$	$2x+7$	$4x+1$

$\begin{bmatrix}-2\\7\end{bmatrix}$ $\begin{bmatrix}-\frac{4}{3}\\\frac{13}{3}\end{bmatrix}$ $\begin{bmatrix}0\\7\end{bmatrix}$ $\begin{bmatrix}3\\13\end{bmatrix}$



ا) $y = |x-2| + |2x+2|$

$R_f = [3, +\infty)$

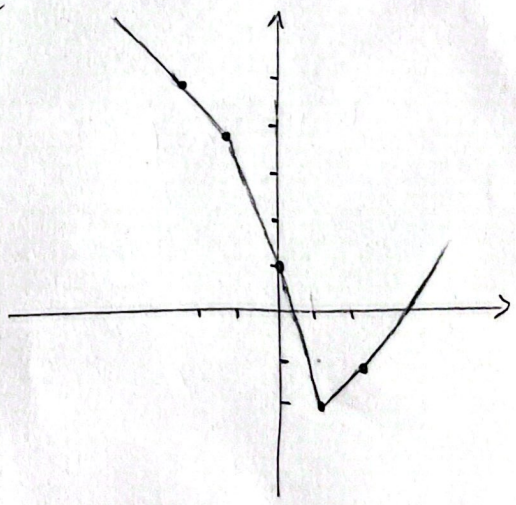


-1	2
$-x+2-2x-2$	$-x+2+2x+2$	$x-2+2x+2$
$-3x$	$x+4$	$3x$

$\begin{bmatrix}-2\\6\end{bmatrix}$ $\begin{bmatrix}-1\\2\end{bmatrix}$ $\begin{bmatrix}0\\4\end{bmatrix}$ $\begin{bmatrix}2\\6\end{bmatrix}$ $\begin{bmatrix}3\\9\end{bmatrix}$

ب) $y = |2x-2| - |x+1|$

$R_f = [-2, +\infty)$



-1	1
$-2x+2+x+1$	$-2x+2-x-1$	$2x-2-x-1$
$-x+3$	$-3x+1$	$x-3$

$\begin{bmatrix}-2\\5\end{bmatrix}$ $\begin{bmatrix}-1\\4\end{bmatrix}$ $\begin{bmatrix}0\\1\end{bmatrix}$ $\begin{bmatrix}1\\-2\end{bmatrix}$ $\begin{bmatrix}2\\-1\end{bmatrix}$