

الف) $y = x^3 - 3x^2 + 3x \Rightarrow y = 1 = (x-1)^3 \Rightarrow x-1 = \sqrt[3]{y-1} \Rightarrow x = \sqrt[3]{y-1} + 1 \Rightarrow R_f = \mathbb{R}$

$\Rightarrow y = \frac{1}{x^2 - 3x} \Rightarrow yx^2 - 3xy - 1 = 0 \Rightarrow \Delta = 9y^2 - (4 \times -1 \times y) = 4y^2 + 4y$

$\Rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} \Rightarrow \frac{+3y \pm \sqrt{4y^2 + 4y}}{2y} \Rightarrow \begin{cases} 4y^2 + 4y \geq 0 \Rightarrow 4y(y+1) \geq 0 \Rightarrow \begin{cases} y=0 \\ y=-1 \end{cases} \\ 2y \neq 0 \Rightarrow y \neq 0 \end{cases}$

$\Rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$

ب) $y = x^2 - 4x + 10 \xrightarrow{a > 0} \begin{cases} x_{min} = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y_{min} = 4 - 16 + 10 = -2 \end{cases} \Rightarrow R_f = [-2, +\infty)$

$\Rightarrow y = -x^2 + 4x + 3 \xrightarrow{a < 0} \begin{cases} x_{max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{max} = -4 + 16 + 3 = 15 \end{cases} \Rightarrow R_f = (-\infty, 15]$

ج) $y = \sqrt{x^2 - 4x - 3} \xrightarrow{a > 0} \begin{cases} x_{min} = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y_{min} = 4 - 16 - 3 = -15 \end{cases} \Rightarrow R_f = \sqrt{[-15, +\infty)} = [0, +\infty)$

د) $y = \sqrt{4x - x^2} \xrightarrow{a < 0} \begin{cases} x_{max} = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_{max} = -4 + 16 = 12 \end{cases} \Rightarrow R_f = \sqrt{(-\infty, 12]} \Rightarrow [0, 4]$

هـ) $y = x^3 - 3x^2 + 3x + 1 \Rightarrow R_f = \mathbb{R}$

و) $y = x^3 - 3x^2 + 3x + 2 \Rightarrow R_f = \mathbb{R}$

ز) $\sqrt{x^3 - 3x^2 + 3x + 1} \Rightarrow R_f = [0, +\infty)$

ح) $(x^3 - 3x^2 + 3x + 1)^2 \Rightarrow R_f = [0, +\infty)$

الف) $y = \frac{3x+1}{x-2} \Rightarrow \begin{cases} c \neq 0 \\ 3x-2 \neq 1 \times 1 \end{cases} \Rightarrow R_f = \mathbb{R} - \{3\}$

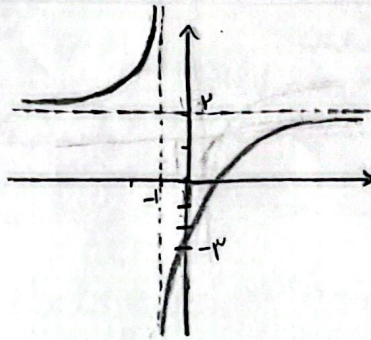
ب) $y = \frac{2x+1}{x+1} \Rightarrow \begin{cases} c \neq 0 \\ 2x+1 \neq 1 \times 1 \end{cases} \Rightarrow R_f = \mathbb{R} - \{2\}$

الف) $y = \sqrt{\frac{x+2}{x+1}} \Rightarrow R_f = [0, +\infty) - \{\sqrt{2}\}$

ب) $\sqrt{\frac{x+1}{x-1}} \Rightarrow \sqrt{\frac{x+1}{-x+1}} \Rightarrow R_f = [0, +\infty) - \{\sqrt{2}\} \Rightarrow R_f = [0, +\infty)$

الف) $y = \frac{x-3}{x+1} \Rightarrow R_f = \mathbb{R} - \{2\}$

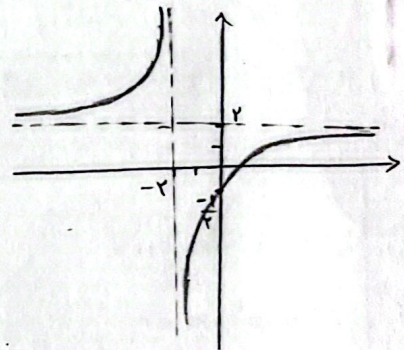
مستقيمات مقاربة:
 $\frac{x-3}{x+1} = -1$
 $\frac{x-3}{x+1} = 2$



نقطة تقاطع المحاور $= \frac{-3}{1} = -3$

ب) $y = \frac{x-1}{x+2} \Rightarrow R_f = \mathbb{R} - \{2\}$

مستقيمات مقاربة:
 $\frac{x-1}{x+2} = -2$
 $\frac{x-1}{x+2} = 2$



الف) $y = \sin x + \frac{1}{\sin x} \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$

ب) $y = \frac{x^4+1}{x^2} \Rightarrow \frac{x^4}{x^2} + \frac{1}{x^2} \Rightarrow x^2 + \frac{1}{x^2} \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$

ج) $\frac{\sqrt[3]{x^2+1}}{\sqrt[3]{x}} \Rightarrow \sqrt[3]{\frac{x^2}{x}} + \frac{1}{\sqrt[3]{x}} \Rightarrow \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \Rightarrow R_f = [2, +\infty)$

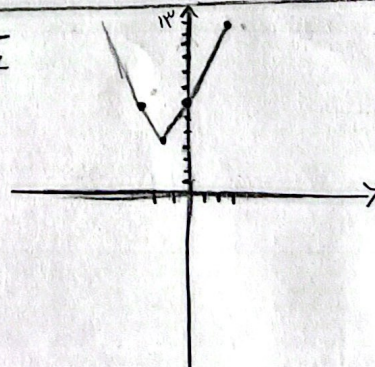
الف) $y = x^2 + \frac{1}{x^2+3} \rightarrow x^2+3 = z \Rightarrow x^2 = z-3 \Rightarrow z + \frac{1}{z} - 3 \Rightarrow R_f = [\frac{1}{4}, +\infty)$

ب) $y = \frac{x^2+2}{\sqrt{x^2+2}} \Rightarrow \frac{x^2+2+1}{\sqrt{x^2+2}} \Rightarrow \sqrt{x^2+2} + \frac{1}{\sqrt{x^2+2}} \Rightarrow R_f = [\frac{3}{2}, +\infty)$

$y = |x-3| + |3x+2| \Rightarrow$

x	$ x-3 $	$ 3x+2 $	y
$x < -\frac{2}{3}$	$3-x$	$-(3x+2)$	$-x-1$
$-\frac{2}{3} < x < 3$	$3-x$	$3x+2$	5
$x > 3$	$x-3$	$3x+2$	$4x-1$

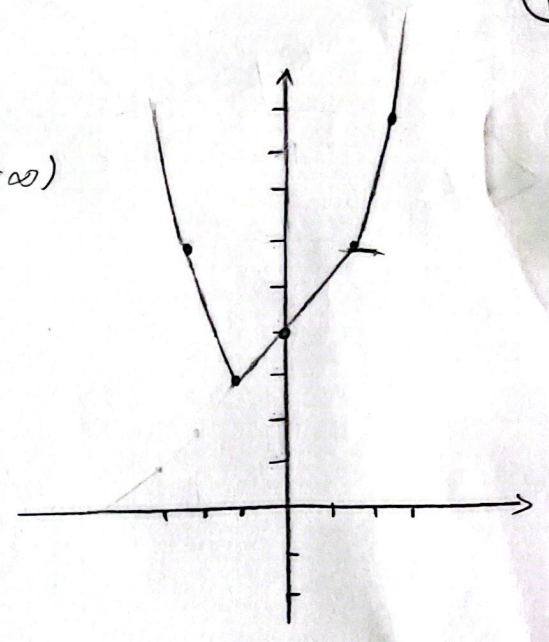
نقاط التقاطع: $-\frac{2}{3}$ و 3



ii) $y = |x - 2| + |2x + 2|$

$R_f = [2, +\infty)$

	-1	2
$-x + 2 + 2x + 2$	$-x + 2 + 2x + 2$	$x - 2 + 2x + 2$
$-x$	$x + 2$	$3x$
$[-x]$	$[-1]$	$[2]$
$[2]$	$[0]$	$[3]$



$\rightarrow y = |2x - 2| - |x + 1|$

$R_f = [2, +\infty)$

	-1	1
$-2x + 2 + x + 1$	$-2x + 2 - x - 1$	$2x - 2 - x - 1$
$-x + 3$	$-3x + 1$	$x - 3$
$[-x]$	$[-1]$	$[1]$
$[3]$	$[1]$	$[-3]$

