

→ ا طهر عارانی

$$y = x^3 - 3x^2 + 3x \quad (5)$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$R \rightarrow a, b=1 \quad (x-1)^3 = x^3 - 3x^2 + 3x - 1$$

↓

$$y = x^3 - 3x^2 + 3x = (x-1)^3 + 1 \rightarrow y-1 = (x-1)^3$$

$$R \rightarrow (-\infty, +\infty)$$

$$\rightarrow y = \frac{1}{x^2 - 2x} \rightarrow \frac{1}{(x-1)^2 - 1}$$

$$x^2 - 2x = (x-1)^2 - 1$$

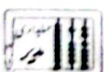
$$y(x-1)^2 - y = 1 \rightarrow y(x-1)^2 = 1+y \rightarrow (x-1)^2 = \frac{1+y}{y}$$

$$\frac{1+y}{y} \geq 0 \rightarrow \frac{1}{y} + \frac{y}{y} \geq 0 \rightarrow \frac{1}{y} + 1 \geq 0 \rightarrow \frac{1}{y} \geq -1$$

$$y \leq -1 \rightarrow y \in (-\infty, -1] \Rightarrow (x-1)^2 - 1 \in [-1, 0) \cup (0, +\infty)$$

↓

$$\rightarrow R \rightarrow (-\infty, -1] \cup (0, +\infty)$$



ا) $x^2 - 8x + 10 \rightarrow \frac{-b}{2a} = \frac{8}{2} = 4 \rightarrow x_s = 4$
 $\hookrightarrow 4 - 8 + 10 = 6 \rightarrow \min$ (5)

$R \rightarrow [6, +\infty)$

ب) $-x^2 + 4x + 5 \rightarrow \frac{-b}{2a} = \frac{-4}{-2} = 2 \rightarrow x_s = 2$
 $\hookrightarrow \max = y_s = -4 + 8 + 5 = 9$

$R \rightarrow (-\infty, 9]$

ج) $\sqrt{x^2 - 8x - 4} \rightarrow \frac{-b}{2a} = \frac{8}{2} = 4 \rightarrow \min = y_s = -4$

$(\sqrt{-4}, +\infty) \rightarrow \sqrt{-4}$

$\hookrightarrow R = [0, +\infty)$

د) $\sqrt{4x - x^2} \rightarrow \sqrt{-x^2 + 4x}$

$x_s \rightarrow 2 \quad \max = 4 - 4 = 0$

القيمة العظمى = 0

$R = [0, 2]$

هـ) $y = x^2 - 8x^2 + 2x + 1 \rightarrow$ " " $\rightarrow R = \mathbb{R}$ (5)

و) $y = x^5 - 4x^4 + 2x + 2 \rightarrow$ " " $\rightarrow R = \mathbb{R}$

ز) $y = \sqrt{x^2 - 4x^2 + 2x + 1} \rightarrow$ " " $\rightarrow R = [0, +\infty)$

ح) $y = (x^2 - 8x^2 + 2x + 1)^2 \rightarrow$ " " $\rightarrow R = [0, +\infty)$
 تكون زوج



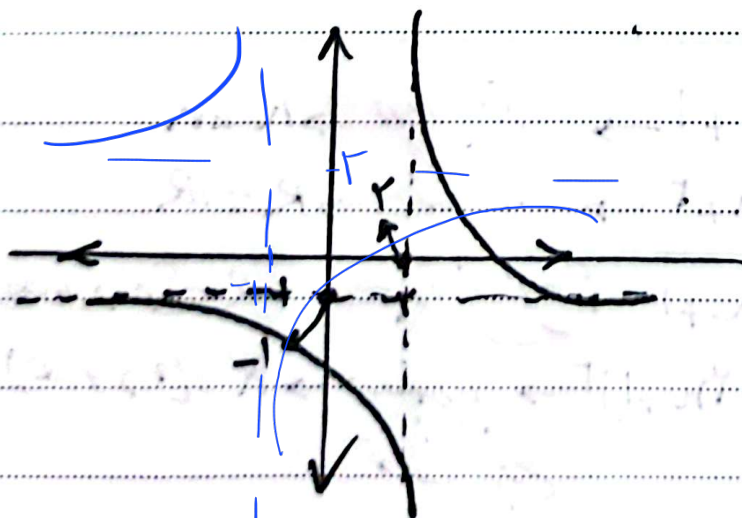
الف) $y = \frac{r_n + 1}{n - r} \rightarrow y(n - r) = r_n + 1 \rightarrow n(y - r) = ry + 1$
 $n = \frac{ry + 1}{y - r} \rightarrow y \neq r \rightarrow R \in \mathbb{R} - \{r\}$

ب) $y = \frac{r_n + 1}{n + 1} \rightarrow y(n + 1) = r_n + 1 \rightarrow y(n - r) = -y + 1$
 $n(y - r) = -y + 1 \rightarrow n = \frac{-y + 1}{y - r}$
 $\hookrightarrow y \neq r \rightarrow R = \mathbb{R} - \{r\}$

الف) $y = \sqrt{\frac{r_n + 1}{n + 1}} \rightarrow R = [0, +\infty) - \{r\}$
 جانب اقصی

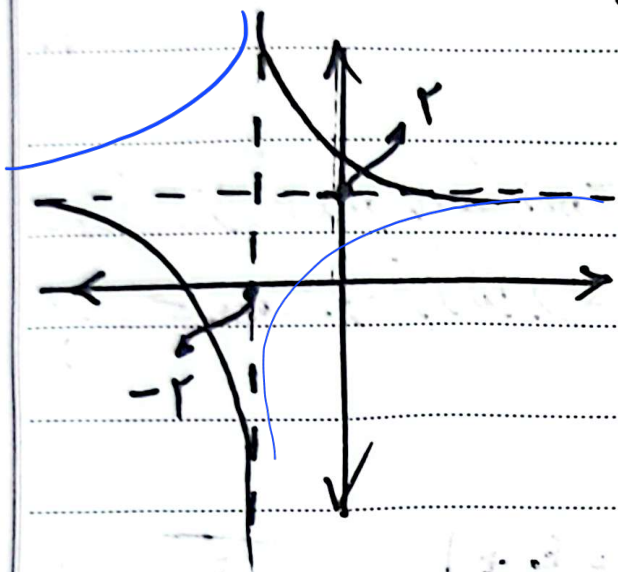
ب) $y = \sqrt{\frac{r_n + 1}{-n + r}} \rightarrow R = [0, +\infty) - \{r\}$

الف) $y = \frac{r_n - r}{n + 1} \rightarrow n + 1 = 0 \rightarrow n = -1$ قاع
 $\hookrightarrow \frac{r}{1} = r$ اقصی



$$\rightarrow) \frac{r n - 1}{n + r} = y \rightarrow n + r = 0 \rightarrow n = -r \quad \text{مقام}$$

$$\rightarrow \frac{r}{1} = r \rightarrow \text{افقی}$$



$$\text{ثاني) } y = \sin n + \frac{1}{\sin n}$$

5

$$\begin{cases} \sin n > 0 \rightarrow R_y = [r, +\infty) \\ \sin n < 0 \rightarrow R_y = (-\infty, -r] \end{cases} \Rightarrow R_y = \textcircled{1} \cup \textcircled{2}$$

$$\rightarrow) y = \frac{n^4 + 1}{n^2} \rightarrow y = \frac{n^4}{n^2} + \frac{1}{n^2} \rightarrow y = n^2 + \frac{1}{n^2}$$

$$R = (-\infty, -r] \cup [r, +\infty)$$

$$\text{ع.) } y = \frac{\sqrt[3]{n^4 + 1}}{\sqrt[3]{n}} \rightarrow y = \frac{\sqrt[3]{n} \cdot \sqrt[3]{n}}{\sqrt[3]{n}} + \frac{1}{\sqrt[3]{n}}$$

$$\hookrightarrow \sqrt[3]{n} + \frac{1}{\sqrt[3]{n}} \rightarrow R = (-\infty, -r] \cup [r, +\infty)$$



$$1) \sqrt{x} + \frac{1}{\sqrt{x}} = y \rightarrow R = [2, +\infty)$$

$$y = x^2 + \frac{1}{x^2 + 3}$$

1,5

-A

حاصل مقدار x^2 دایره با صفر است پس $\min$ محبت $\rightarrow \min$
 $\frac{1}{x^2 + 3}$ و محدودیت و خورندار چون $x^2 \neq 3$

$$Ry = [\frac{1}{4}, +\infty)$$

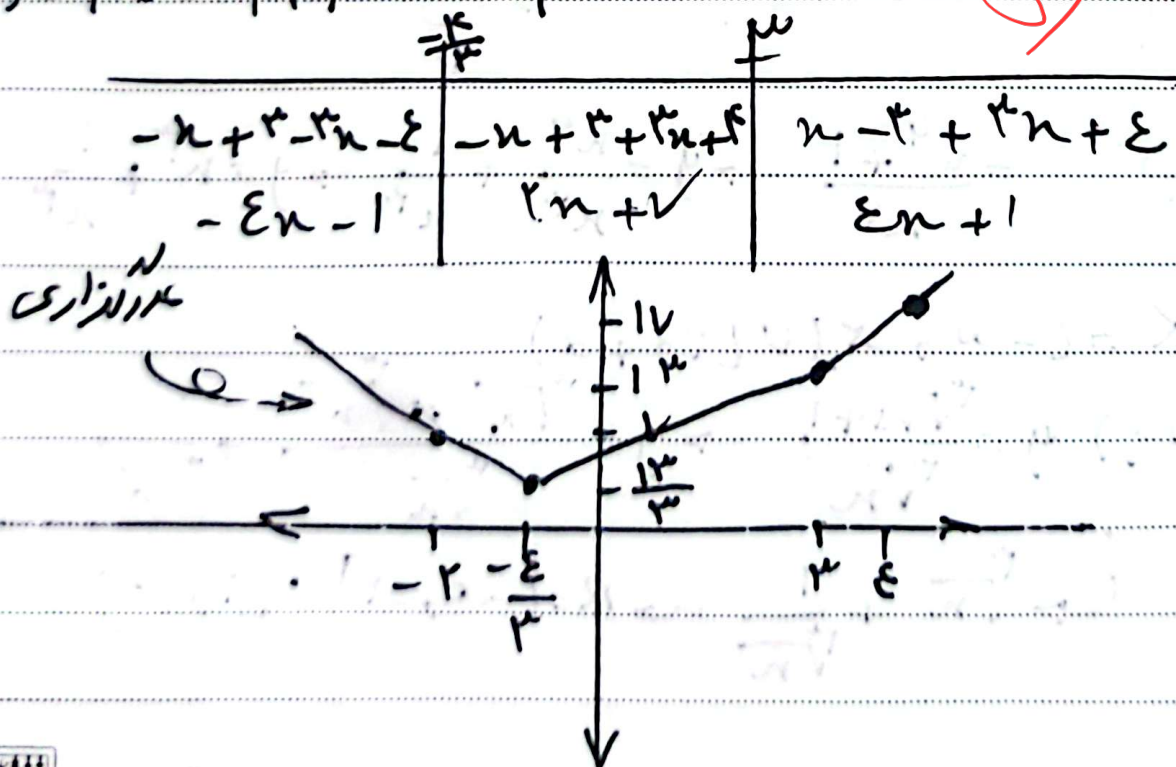
$$b) y = \frac{x^2 + 1}{\sqrt{x^2 + 4}} \rightarrow \frac{x^2 + \epsilon + 1}{\sqrt{x^2 + \epsilon}} \rightarrow \sqrt{x^2 + \epsilon} + \frac{1}{\sqrt{x^2 + \epsilon}}$$

$$\hookrightarrow R = [2, +\infty)$$

$$y = |x - 2| + |3x + \epsilon|$$

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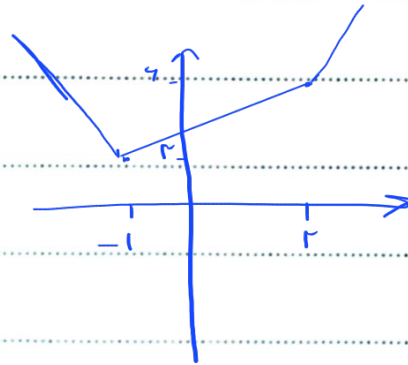
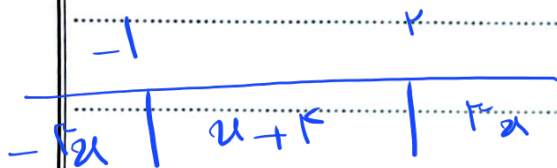
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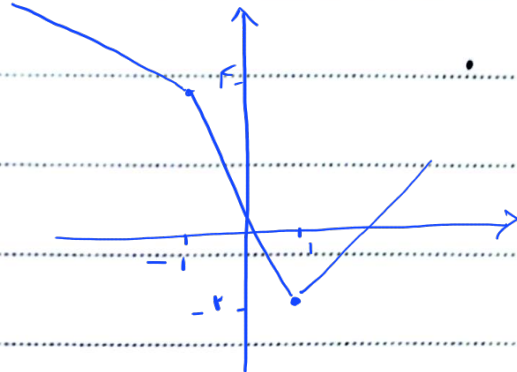
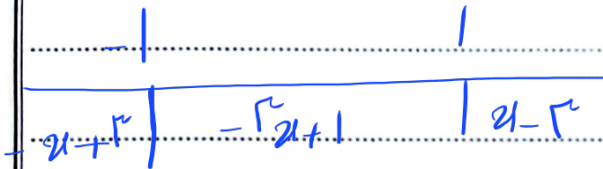
الف) $y = |x-2| + |2x+2| \rightarrow R = [4, +\infty)$ (10)
 (فصل سانی)

ب) $y = |2x-2| - |x+1| \rightarrow R = [-4, -2]$
 (فصل آبی)

$\sqrt{2x^2+x} + \frac{1}{\sqrt{2x^2+x}} \xrightarrow{x^2=0} \frac{x}{x} \quad R = [\frac{1}{2}, +\infty)$



$R = [2, +\infty)$



$R = [-1, +\infty)$

