

الف) $y = x^3 - 3x^2 + 3x - 1 + 1 = (x-1)^3 + 1 \rightarrow y-1 = (x-1)^3 \Rightarrow x-1 = \sqrt[3]{y-1} \Rightarrow x = \sqrt[3]{y-1} + 1$

$\rightarrow R_f = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \rightarrow yx^2 - 2xy = 1 \rightarrow yx^2 - 2xy - 1 = 0 \quad \Delta = 4y^2 + 4y \quad x = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y}$

$4y^2 + 4y \geq 0 \rightarrow y^2 + y \geq 0 \Rightarrow y(y+1) \geq 0 \quad \frac{-1}{+ \quad - \quad +} \rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$

الف) $y = x^2 - 4x + 4 \xrightarrow{a.f.} \begin{cases} x_{\min} = \frac{-b}{2a} = 2 \\ y_{\min} = 0 \end{cases}$

$R_f = [0, +\infty)$

ب) $y = -x^2 + 4x + 3 \xrightarrow{a.f.} \begin{cases} x_{\max} = \frac{-b}{2a} = 2 \\ y_{\max} = 11 \end{cases}$

$R_f = (-\infty, 11]$

ج) $y = \sqrt{x^2 - 4x + 4} \xrightarrow{a.f.} \begin{cases} x_{\min} = \frac{-b}{2a} = 2 \\ y_{\min} = 0 \end{cases}$

$R_f = \sqrt{[-1, +\infty)} = [0, +\infty)$

د) $y = \sqrt{-x^2 + 4x} \xrightarrow{a.f.} \begin{cases} x_{\max} = \frac{-b}{2a} = 2 \\ y_{\max} = 2 \end{cases}$

$R_f = \sqrt{[-\infty, 4]} = [0, 2]$

الف) $y = x^3 - 3x^2 + 3x + 1 \rightarrow R_f = \mathbb{R}$

ب) $y = x^3 - 4x^2 + 4x + 2 \rightarrow R_f = \mathbb{R}$

ج) $y = \sqrt{x^3 - 4x^2 + 4x + 1} \rightarrow R_f = \sqrt{\mathbb{R}} = [0, +\infty)$

د) $y = (x^3 - 4x^2 + 4x + 1)^2 \rightarrow R_f = (\mathbb{R})^2 = [0, +\infty)$

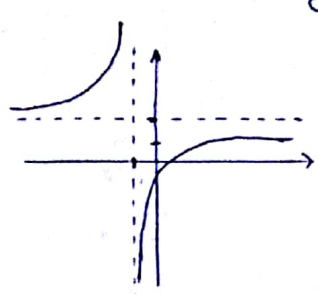
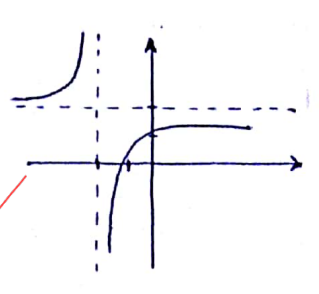
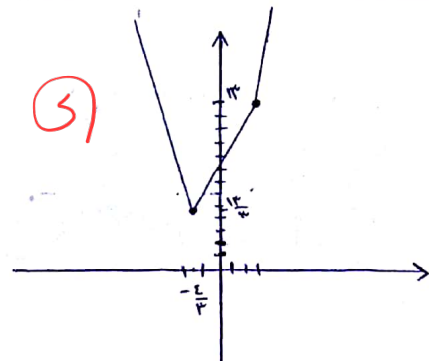
الف) $y = \frac{x+1}{x-2} \rightarrow a=1, b=1, c=1, d=-2 \quad R_f = \mathbb{R} - \left\{\frac{a}{c}\right\} \rightarrow R_f = \mathbb{R} - \{1\}$

ب) $y = \frac{x+1}{x+1} \rightarrow a=1, b=1, c=1, d=1 \quad R_f = \mathbb{R} - \left\{\frac{a}{c}\right\} \rightarrow R_f = \mathbb{R} - \{1\}$

الف) $y = \sqrt{\frac{x+1}{x+1}} \rightarrow \sqrt{\mathbb{R} - \{1\}} \Rightarrow R_f = [0, +\infty) - \{1\}$

$R_f = \mathbb{R} - \left\{\frac{a}{c}\right\}$ ← تابع صورتی

ب) $y = \sqrt{\frac{x+1}{-x+1}} \rightarrow \sqrt{\mathbb{R} - \{1\}} \Rightarrow R_f = [0, +\infty)$

ا) $y = \frac{x-1}{x+1}$ جانب $x=1$ = $\frac{0}{2} = 0$ جانب $x = -1$ = $\frac{2}{0}$ 	ب) $y = \frac{x-1}{x+1}$ جانب $x=1$ = $\frac{0}{2} = 0$ جانب $x = -1$ = $\frac{2}{0}$ 	6
ا) $y = \sin x + \frac{1}{\sin x} \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$ ب) $y = \frac{x^4+1}{x^2} = x^2 + \frac{1}{x^2} \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$ ج) $y = \frac{\sqrt{x^2+1}}{\sqrt{x}} = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$ د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow R_f = [1, +\infty)$		7
ا) $y = x^2 + \frac{1}{x^2+x} \xrightarrow{x \rightarrow -\infty} x^2+x + \frac{1}{x^2+x} - x^2 \rightarrow R_f = [\frac{1}{4}, +\infty)$ ب) $y = \frac{x^2+a}{\sqrt{x^2+2}} = \sqrt{x^2+2} + \frac{1}{\sqrt{x^2+2}} \rightarrow R_f = [\frac{a}{2}, +\infty)$		8
$y = x-1 + x+2$ $x=1$ $x=-2$ $-x+1-x-2 = -x-1$ $-x+1+x+2 = x+3$ $x-1+x+2 = x+1$ $\frac{10}{4} = 2.5$ $\frac{14}{4} = 3.5$		9
ا) $y = x-1 + x+2$ $-x+1-x-2 = -x-1$ $-x+1+x+2 = x+3$ $x-1+x+2 = x+1$ $\frac{10}{4} = 2.5$ $\frac{14}{4} = 3.5$ $R_f = [2.5, +\infty)$	ب) $y = x-1 - x+1$ $-x+1-x-1 = -x-2$ $-x+1+x+1 = x+2$ $x-1+x-1 = x-2$ $\frac{10}{4} = 2.5$ $\frac{14}{4} = 3.5$ $R_f = [-2, +\infty)$	10