۱۸٫۲۵

۱)

الف) $y = x^3 - 3x^2 + 3x + 1 - 1$

$y = (x-1)^3 + 1$

$y - 1 = (x-1)^3 \Rightarrow (y-1)^{\frac{1}{3}} = x - 1 \Rightarrow x = (y-1)^{\frac{1}{3}} + 1 \Rightarrow R = \mathbb{R}$

د

ب) $y = \dfrac{1}{x^2 - 2x}$ $\quad x^2 - 2x \neq 0 \rightarrow x(x-2) \neq 0 \rightarrow x \neq 0,\ x \neq 2$

$yx^2 - 2yx = 1 \rightarrow yx^2 - 2yx - 1 = 0 \quad \Delta = 4y^2 - 4 \times y \times -1 = 4y^2 + 4y \geq 0$

$4y(y+1) \geq 0$ $\quad$ ۱ $\quad$ ۰

$R = (-\infty, -1] \cup (0, \infty)$ $\quad + \ \ - \ \ +$

$x \neq 0 \Leftarrow$

۲)

الف) $y = x^2 - 4x + 10$

$x_s = \dfrac{-b}{2a}$, $y_s = \dfrac{-\Delta}{4a} \Rightarrow \left| \dfrac{4}{2} = 2,\ 6 \right.$

$R = [6, +\infty)$

ب) $y = -x^2 + 6x + 3$

| ۳, ۱۲

$R = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 3}$

| ۲, −۷

$\sqrt{[-7, +\infty)}$

$R = [0, +\infty)$

د

د) $y = \sqrt{6x - x^2}$

| ۳, ۹

$\sqrt{(-\infty, 9]}$

$R = [0, 3]$

۳)

الف) $y = x^3 - 5x^2 + 2x + 1$

$R = \mathbb{R}$

ب) $y = x^7 - 6x^4 + 3x + 2$

$R = \mathbb{R}$

د

ج) $y = \sqrt{x^7 - 6x^2 + 5x + 1}$

$x^7 - 6x^2 + 5x + 1$

$R = \mathbb{R} \Rightarrow \sqrt{\mathbb{R}} = [0, +\infty)$

$R = [0, +\infty)$

د) $y = (x^5 - 4x^2 + 3x + 1)^2$

$x^5 - 4x^2 + 3x + 1$

$R = \mathbb{R}$

$R_y = [0, +\infty)$

۴)

الف) $y = \dfrac{3x+1}{x-2}$

هموگرافیک $\Rightarrow ad \neq cb,\ c \neq 0$

$-6 \neq 1,\ 1 \neq 0$

$R = \mathbb{R} - \{3\}$

ب) $y = \dfrac{2x+1}{x+1}$

$R = \mathbb{R} - \{2\}$

د

(۵) ۲

الف) $y = \dfrac{\sqrt{2}\,x + 4}{x+1}$

$y = \dfrac{2x+4}{x+1} \rightarrow yx + y = 2x + 4$

$\rightarrow yx - 2x = 4 - y \rightarrow x(y-2) = 4 - y \rightarrow x = \dfrac{4-y}{y-2}$

$\mathbb{R} - \{2\} \leftarrow y \neq 2$

$\Rightarrow y = \sqrt{\mathbb{R} - \{2\}} \Rightarrow \mathbb{R} = [0, +\infty) - \{\sqrt{2}\}$

ب) $y = \dfrac{\sqrt{3x+1}}{2-x}$

$y = \dfrac{3x+1}{2-x} \rightarrow x = \dfrac{-2y+1}{-y-3}$

$\mathbb{R} - \{-3\} \quad y \neq -3 \leftarrow$

$\sqrt{\mathbb{R} - \{-3\}} = [0, +\infty) - \{\sqrt{-3}\}$

$\boxed{\mathbb{R} = [0, +\infty)}$ $\quad \sqrt{-3}$ ⇓ نت

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(۶) ۲

الف) $y = \dfrac{2x-3}{x+1}$

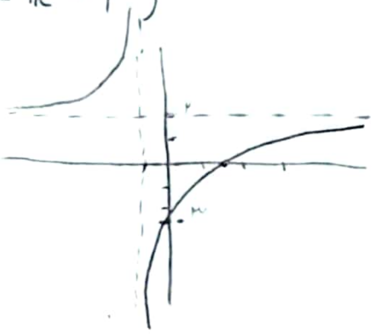
$R = \mathbb{R} - \{2\}$

مجانب قائم $= -1$

مجانب افقی $= 2$

$x = 0 \rightarrow y = -3$

$y = 0 \rightarrow 2x = 3 \rightarrow x = \frac{3}{2}$



ب) $y = \dfrac{2x-1}{x+2}$

مجانب قائم $= -2$

مجانب افقی $= 2$

$x = 0 \rightarrow y = -\frac{1}{2}$

$R = \mathbb{R} - \{2\}$

---

(۷) ۲

الف) $y = \sin x + \dfrac{1}{\sin x}$

$a + \frac{1}{a}$

$-1 \leq \sin x \leq 1$

$-1 + (-\frac{1}{1}) = -2$

$R = (-\infty, -2] \cup [2, +\infty)$

ب) $y = \dfrac{x^4 + 1}{x^2}$

$x^2 + \frac{1}{x^2}$

$R = (-\infty, -2] \cup [2, +\infty)$

ج) $y = \dfrac{\sqrt[3]{x^2} + 1}{\sqrt[3]{x}}$

$\sqrt[3]{x} + \dfrac{1}{\sqrt[3]{x}} \quad R = (-\infty, -2] \cup [2, +\infty)$

د) $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$

$R = [2, +\infty)$

---

(۸) ۱,۵

الف) $y = x^2 + \dfrac{1}{x^2 + 3}$

$x^2 + 3 - 3 + \dfrac{1}{x^2 + 3} \quad R = [\frac{1}{3}, +\infty)$

$\Rightarrow (x^2 + 3) + \dfrac{1}{(x^2 + 3)} - 3$

$x_{min} = 0 \rightarrow 3 + \frac{1}{3} - 3 = \frac{1}{3}$

ب) $y = \dfrac{x^2 + 5}{\sqrt{x^2 + 4}}$

$\dfrac{x^2 + 4 + 1}{\sqrt{x^2 + 4}}$

$\Rightarrow \sqrt{x^2 + 4} + \dfrac{1}{\sqrt{x^2+4}}$

$x = 0 \rightarrow \sqrt{4} = 2$

$R = [2, +\infty)$

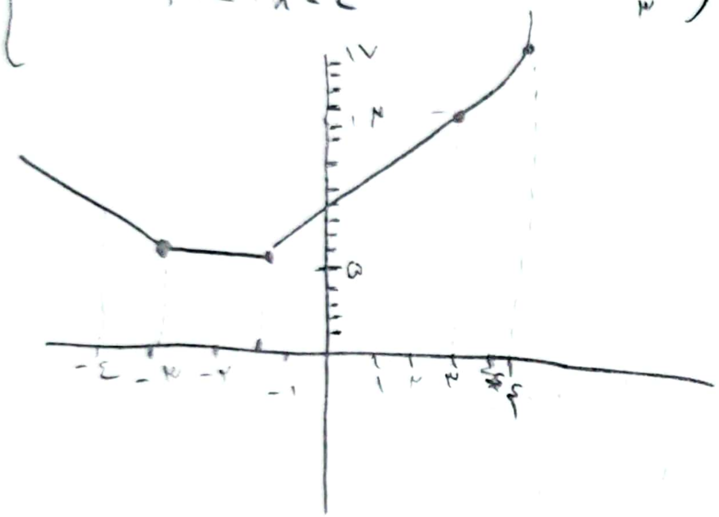
(9)

$y = |x-3| + |3x+4|$

$$\begin{cases} x-3+3x+4 & x \geq 3 \\ -x+3+3x+4 & -\frac{4}{3} \leq x \leq 3 \\ -x+3-3x-4 & x \leq -\frac{4}{3} \end{cases} \rightarrow \begin{cases} 4x+1 & x \geq 3 \\ 2x+7 & -\frac{4}{3} \leq x \leq 3 \\ -4x-1 & x \leq -\frac{4}{3} \end{cases}$$

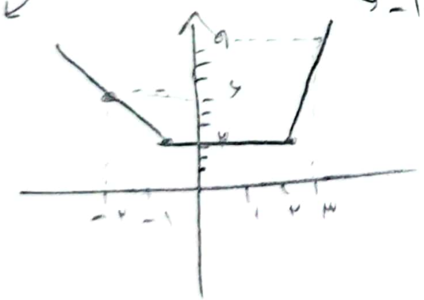
$R = [\frac{13}{3}, +\infty)$

۵



(10)

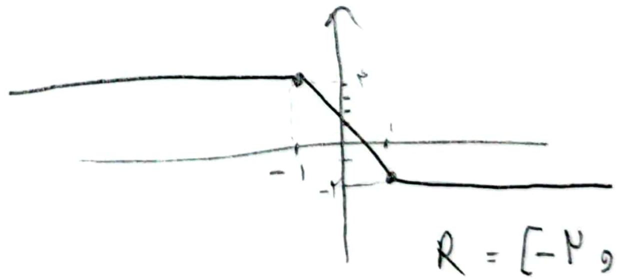
الف) $y = |x-2| + |2x+2|$



$R = [3, +\infty)$

۰/۷۵

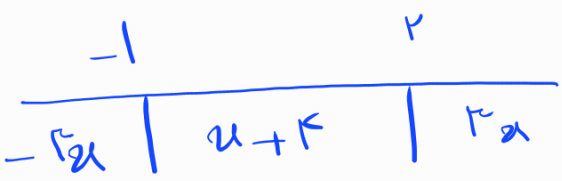
ب) $y = |2x-2| - |x+1|$



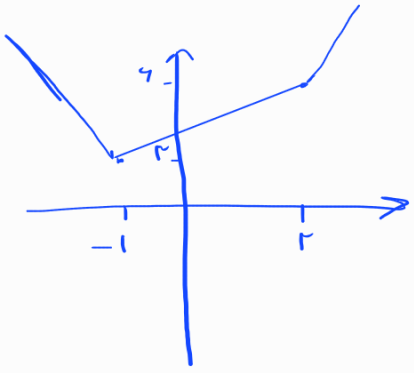
$R = [-2, 2]$

$\sqrt{2x^2+3} + \frac{1}{\sqrt{2x^2+4}}$ $\xrightarrow{x^2=0}$ $\frac{3}{2}$ $\quad R=[\frac{3}{2}, +\infty)$ (ب ۸

(الف ۱۰



$R=[2, +\infty)$



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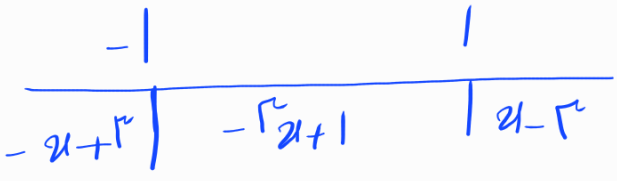

$R=[-2, +\infty)$