۱۹

آکوستا یادی یازدهم دفتر A تکلیف شماره ۶

۱-

الف) $y = x^3 - 3x^2 + 3x \rightarrow y = (x-1)^3 + 1 \rightarrow (x-1)^3 = y - 1 \rightarrow x - 1 = \sqrt[3]{y-1}$

$\Rightarrow x = \sqrt[3]{y-1} + 1 \rightarrow R_f = \mathbb{R}$

ب) $y = \dfrac{1}{x^2 - 2x} \rightarrow yx^2 - 2yx = 1 \rightarrow yx^2 - 2yx - 1 = 0 \rightarrow x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$

$\rightarrow x = \dfrac{2y \pm \sqrt{4y^2 + 4y}}{2y} \rightarrow \Delta \geq 0 \rightarrow 4y^2 + 4y \geq 0 \rightarrow y(4y+4) \geq 0$

x		-1		0	
	$+$	0	$-$	0	$+$

$R_f = (-\infty, -1] \cup [0, +\infty)$

۲-

الف) $y = x^2 - 4x + 10 \xrightarrow{a>0} \begin{cases} x_{min} = \dfrac{-b}{2a} = 2 \\ y_{min} = 6 \end{cases} \rightarrow R_f = [6, +\infty)$

ب) $y = -x^2 + 6x + 3 \xrightarrow{a<0} \begin{cases} x_{max} = \dfrac{-b}{2a} = 3 \\ y_{max} = 12 \end{cases} \rightarrow R_f = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 3} \xrightarrow{a>0} \begin{cases} x_{min} = \dfrac{-b}{2a} = 2 \\ y_{min} = -7 \end{cases} \rightarrow R_f = \sqrt{[-7, +\infty)} = [0, +\infty)$

د) $y = \sqrt{6x - x^2} \xrightarrow{a<0} \begin{cases} x_{max} = \dfrac{-b}{2a} = 3 \\ y_{max} = 9 \end{cases} \rightarrow R_f = \sqrt{(-\infty, 9]} = [0, 3]$

۳-

الف) $x^3 - 5x^2 + 2x + 1 = y \qquad R_f = \mathbb{R}$

ب) $y = x^7 - 6x^4 + 3x + 2 \qquad R_f = \mathbb{R}$

ج) $y = \sqrt{x^7 - 6x^4 + 5x + 1} \qquad R_f = \sqrt{\mathbb{R}} = [0, +\infty)$

د) $y = (x^5 - 4x^2 + 3x + 1)^2 \qquad R_f = [0, +\infty)$

s.a.m

۴ـ
الف) $y=\frac{3x+1}{x-2}$ $\quad R_f=\mathbb{R}-\{\frac{a}{c}\}=\mathbb{R}-\{3\}$

ب) $y=\frac{2x+1}{x+1}$ $\quad R_f=\mathbb{R}-\{\frac{a}{c}\}=\mathbb{R}-\{2\}$

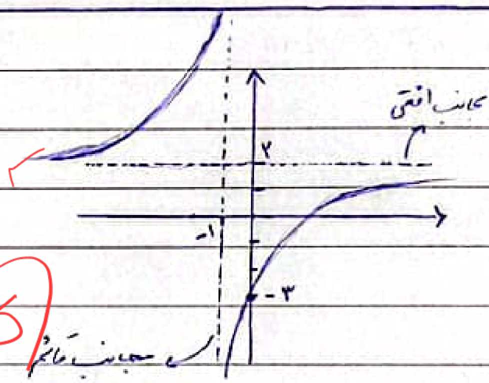
۵ـ
الف) $y=\sqrt{\frac{2x+4}{x+1}}$ $\quad R_f=\sqrt{\mathbb{R}-\{2\}}=[0,+\infty)-\{+\sqrt{2}\}$

ب) $y=\sqrt{\frac{3x+1}{2-x}}$ $\quad R_f=\sqrt{\mathbb{R}-\{-3\}}=[0,+\infty)$

۶ـ
الف) $y=\frac{2x-3}{x+1} \longrightarrow$ مجانب قائم ← ریشه مخرج = $-1=x$

مجانب افقی ← $\frac{a}{c}=2=y$

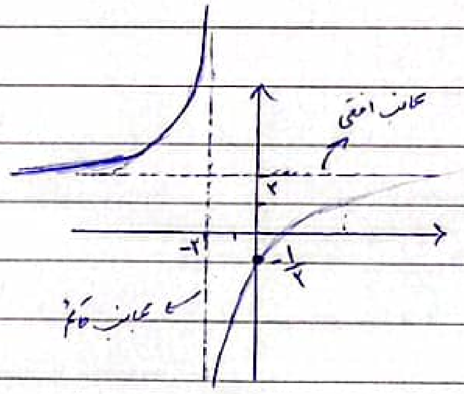
$x=0 \longrightarrow y=-3 \longrightarrow \begin{vmatrix}0\\-3\end{vmatrix}$



ب) $y=\frac{2x-1}{x+2} \longrightarrow$ مجانب قائم ← $-2=x$

مجانب افقی ← $2=y$

$x=0 \longrightarrow y=-\frac{1}{2} \longrightarrow \begin{vmatrix}0\\-\frac{1}{2}\end{vmatrix}$



۷ـ
الف) $\sin x+\frac{1}{\sin x}=y$ $\quad R_f=(-\infty,-2]\cup[2,+\infty)$

ب) $y=\frac{x^4+1}{x^2}=x^2+\frac{1}{x^2}$ $\quad R_f=(-\infty,-2]\cup[2,+\infty)$

ج) $y=\frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}}=\sqrt[3]{x}+\frac{1}{\sqrt[3]{x}}$ $\quad R_f=(-\infty,-2]\cup[2,+\infty)$

د) $\sqrt{x}+\frac{1}{\sqrt{x}}=y$ $\quad R_f=[2,+\infty)$

s.a.m

۸- الف) $y = x^2 + \frac{1}{x^2+3}$ $\quad x^2 \geq 0 \Rightarrow$ ماکزیمم برد $= +\infty$

$n = x^2+3 \quad n \geq 3 \Rightarrow y = (n-3) + \frac{1}{n} = n + \frac{1}{n} - 3 \Rightarrow y_{min} = 3 + \frac{1}{3} - 3 = \frac{1}{3}$

$\Rightarrow R_f = [\frac{1}{3}, +\infty)$

۱٫۵

ب) $y = \frac{x^2+5}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}}$ کمترین مقدار $\sqrt{x^2+4} = \sqrt{0+4} = \sqrt{4} = 2$

$\Rightarrow R_f = [2, +\infty)$ $\quad$ $x^2 = 0$ $\quad \frac{1}{2} + 2 = \frac{5}{2}$ $\quad R = [2.5, +\infty)$

۹- $y = |x-3| + |3x+4|$

$$\begin{cases} x-3+3x+4 & x > 3 \\ -x+3+3x+4 & -\frac{4}{3} \leq x \leq 3 \\ -x+3-3x-4 & x < -\frac{4}{3} \end{cases} \Rightarrow \begin{cases} 4x+1 & x > 3 \\ 2x+7 & -\frac{4}{3} \leq x \leq 3 \\ -4x-1 & x < -\frac{4}{3} \end{cases}$$

$\begin{array}{c|} -2 \\ 7 \end{array} \quad \begin{array}{c|} -\frac{4}{3} \\ \frac{13}{3} \end{array} \quad \begin{array}{c|} 3 \\ 13 \end{array} \quad \begin{array}{c|} 4 \\ 17 \end{array}$

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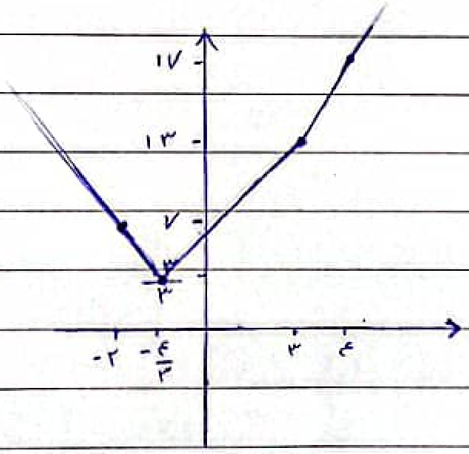
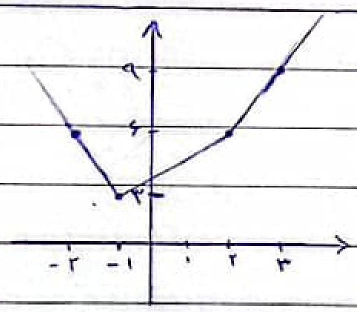


۱۰- الف) $y = |x-2| + |2x+2| \longrightarrow$ $\begin{array}{c|} -2 \\ 6 \end{array} \quad \begin{array}{c|} -1 \\ 3 \end{array} \quad \begin{array}{c|} 2 \\ 6 \end{array} \quad \begin{array}{c|} 3 \\ 9 \end{array}$



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$R = [3, +\infty)$

ب) $y = |2x-2| - |x+1| \longrightarrow$ $\begin{array}{c|} -2 \\ 5 \end{array} \quad \begin{array}{c|} -1 \\ 4 \end{array} \quad \begin{array}{c|} 1 \\ -2 \end{array} \quad \begin{array}{c|} 2 \\ -1 \end{array}$

نوشتن برد

$R = [-2, +\infty)$