

سوال ۱)

$$f(x) = \begin{cases} \cot \frac{\pi x}{x} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases} \quad f \circ f\left(\frac{1}{x}\right)$$

$$\sqrt{x} \rightarrow f(\sqrt{x}) = \sqrt{x^2 + 1} = \sqrt{4} = 2$$

سوال ۲)

$$f\left(\frac{1}{x}\right) = \sqrt{\frac{x-1}{x^2}} \quad g(x) = 2 \cos^2 x \quad f \circ g\left(\frac{\pi}{3}\right) = ?$$

$$\frac{1}{x} = 2 \quad \frac{x}{x} = \frac{1}{x^2} \rightarrow 1 - \frac{1}{x^2} = 1 - \frac{1}{4} = \frac{3}{4} \quad \frac{\pi}{3} = \frac{1}{x} \quad 2 \times \frac{1}{x^2} = \frac{1}{x}$$

$$f \circ g\left(\frac{\pi}{3}\right) = \sqrt{2 \times \frac{1}{x} - \frac{1}{x}} = \sqrt{\frac{1}{x}} = \frac{\sqrt{3}}{2}$$

سوال ۳)

$$f(x) = [x] \quad g(x) = \frac{x}{1-x} \quad f \circ g(\sqrt{2}) = ?$$

$$g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = 2-\sqrt{2} \quad f(-2-\sqrt{2}) = [-2-\sqrt{2}] = -3$$

سوال ۴)

$$f(x) = \sin x \quad g(x) = x \sqrt{1-x^2} \quad g \circ f\left(\frac{\pi}{6}\right) = ?$$

$$f\left(\frac{\pi}{6}\right) = \sin \frac{\pi}{6} = \frac{\sqrt{2}}{2} \quad g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$f(x) = \{(x, \omega), (y, \omega), (1, 12), (10, 12)\}$$

$$g(x) = \{(x, 4), (y, 4), (y, 11), (1, 10)\}$$

الف) $f \circ g(x) \Rightarrow$ $n = x \rightarrow g = 4 \rightarrow f = \omega$ $n = y \rightarrow g = 4 \rightarrow f = \omega$ $n = 7 \rightarrow g = 11 \rightarrow f = 12$
 $n = 1 \rightarrow g = 10 \rightarrow f = 12$ $\{(x, \omega), (y, \omega), (y, 12), (1, 12)\}$

ب) $g \circ f(x) = \emptyset$

ج) $f \circ f(x) = \emptyset$

د) $g \circ g(x)$ $n = x \rightarrow g = 4 \rightarrow 1$ $n = y \rightarrow g = 4 \rightarrow 1$ $n = 7 \rightarrow g = 11 \rightarrow 10$ $\{(x, 1), (y, 1), (y, 4)\}$

$$f = \{(1,1), (3,1), (5,1), (1,5)\}$$

(سوال ۱)

$$g = \{(1,2), (3,1), (a,3), (b,1)\}$$

$$(f,1) \in f \circ g \quad (f,1) \in g \circ f \quad (a,b) = (f,1) \in f(x)$$

$$f(g(x)) = 1$$

$$g(f(x)) = 1$$

$$g(x) = 1$$

$$f(f) = 1$$

$$g(a) = 3 \rightarrow a = f$$

$$g(b) = 1$$

$$b = 1$$

$$g(n+3) = n-1 \rightarrow f = n+3 \quad n = \frac{f-3}{1} \quad 1 \times \left(\frac{f-3}{1}\right) - 1 \Rightarrow f-4$$

(سوال ۲)

$$f(f(x)) = n+3 \rightarrow f(an+b) = an+b \rightarrow a(an+b)+b$$

$$a^2n + ba + b = n+3$$

$$a = \pm 1 \rightarrow b(a+1) = 3$$

$$f(-1) = \begin{cases} -1x-1-1 = -1 \\ 1x-1+1 = 0 \end{cases}$$

$$b(3) = 3 \quad b(-1+1) = 0$$

$$b = 1 \quad b = -3$$

$$g(-1) = 3 + -4 = -1$$

$$f(x) = -1n-1 \quad 1n+1$$

$$f(x) = \sqrt{x+|x|} \quad g(x) = \frac{1}{x-1} \quad g \circ f \Rightarrow D?$$

(سوال ۳)

$$x \in D_f(x)$$

$$x(x-1)$$

$$f(x) \in g(x) \quad f(x) \rightarrow x+|x| \geq 0 \rightarrow x \geq -|x| \rightarrow D_f \subseteq \mathbb{R}$$

$$D_f(x) = \begin{matrix} x=0 \\ x < 1 \end{matrix} \quad D_{g(x)} = \mathbb{R} - \{0, 1\} \quad D_{g \circ f} = \mathbb{R} - \{0, 1\}$$

$$f(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x} \quad (f+g) \circ f \Rightarrow D?$$

(سوال ۴)

$$1-x^2 \geq 0 \rightarrow 1 \geq x^2 \rightarrow -1 \leq x \leq 1 \quad x \geq 0 \quad f+g = \mathbb{Q} \cap \mathbb{R} = D = [0, 1]$$

$$x \in D_f(x) \quad f(x) \in D_{(f+g)} \rightarrow D_{(f+g) \circ f} = [0, 1]$$

Arman

برال شرفی میں نام داس A

Subject:

Year:

Month:

Day:

page: ()

الف) $f\left(\frac{n+1}{x-2}\right) = x+2$

سوال (9)

$$\frac{n+1}{n-2} = t \Rightarrow n+1 = t(n-2)$$

$$n+1 = tn - 2t$$

$$n(1-t) = -2t-1$$

$$n = \frac{-2t-1}{1-t} = \frac{2t+1}{t-1}$$

$$x \frac{2t+1}{t-1} + 2$$

$$\frac{2t+1}{t-1} + \frac{2(t-1)}{t-1} = \frac{2t+1+2t-2}{t-1} = \frac{4t-1}{t-1}$$

ب) $f\left(x + \frac{1}{x}\right) = n^3 \Rightarrow \left(x + \frac{1}{x}\right)^3 = n^3$

$$x + \frac{1}{x} = t$$

$$\left(t\right)^3 = t^3 + \frac{1}{t^3} + 3t \Rightarrow t^3 - 3t = \frac{1}{t^3}$$

$$n^3 - 3n = f(x)$$

9(x) = a(n-1)(n-2\sqrt{2})

سوال (10)

$$f(x) = x\sqrt{x}$$

$$f(1) = 0 \quad f(\sqrt{2}) = 0$$

$$x\sqrt{x} = 1$$

$$x\sqrt{x} = 2\sqrt{2}$$

$$x = 1$$

$$\Rightarrow \underline{2-1=1}$$