(۱)
$$f\left(\frac{2}{3}\right) = \cot\frac{\pi}{6} = \sqrt{3} \rightarrow f(\sqrt{3}) = \sqrt{3+1} = 2$$

(۲)

الف)
$$f\left(\frac{1}{x}\right) = \sqrt{\frac{2x-1}{x^2}} \rightarrow g(x) = 2\cos^2 x \rightarrow g\left(\frac{\pi}{3}\right) = 2\cos^2\frac{\pi}{3} = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$$

ب)
$$g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} = -\sqrt{2}-2 \rightarrow f(-\sqrt{2}-2) = [-\sqrt{2}-2] = -4$$

(۳)
$$f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \rightarrow g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}} = \sqrt{\frac{2}{4}} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{1}{2}$$

(۴)

الف)
$$4 \xrightarrow{g} 6 \xrightarrow{f} 5$$
$$2 \rightarrow 4 \rightarrow 5 \rightarrow fog = \{(4,5),(2,5),(6,12),(8,12)\}$$
$$6 \rightarrow 8 \rightarrow 12$$
$$8 \rightarrow 10 \rightarrow 14$$

ب) $\varnothing$ ج) $\varnothing$

د)
$$4 \xrightarrow{g} 6 \xrightarrow{g} 8$$
$$2 \rightarrow 4 \rightarrow 6 \quad gog = \{(4,8),(2,6),(6,10)\}$$
$$6 \rightarrow 8 \rightarrow 10$$

(۵)
$$(4,1) \in gof \Rightarrow 4 \xrightarrow{f} 5 \xrightarrow{g} 1 \overset{\text{پس}}{\Rightarrow} b = 5$$
$$(4,2) \in fog \Rightarrow 4 \xrightarrow{g} 3 \xrightarrow{f} 2 \Rightarrow a = 4 \rightarrow (a,b) \rightarrow (4,5)$$

(۶)
$$g(2x+3) = 3x-2 \rightarrow 2x+3 = t \rightarrow x = \frac{t-3}{2} \rightarrow g(x) = \frac{3x-13}{2}$$

$$f(ax+b) = a^2x+ab+b = 4x+3 \rightarrow \begin{cases} a^2 = 4 \rightarrow a = \pm 2 \\ b(a+1) = 3 \end{cases} \quad \begin{matrix} a = 2 \rightarrow b = 1 \\ a = -2 \rightarrow b = -3 \end{matrix}$$

$$f(x) = 2x+1 \xrightarrow{f(-1)} -1 \rightarrow g(-1) = -8$$
$$f(x) = -2x-3 \xrightarrow{f(-1)} -1$$

(۷)
$$f\begin{cases} 0<x & 2x \\ x \leqslant 0 & 0 \end{cases} \quad D_f = \mathbb{R} \quad D_g = \mathbb{R}-\{0,4\} \quad D_{gof} = \{x \in \mathbb{R} : \sqrt{x+|x|} \neq \{0,4\}\}$$

$$\sqrt{x+|x|} = 0 \quad D_{gof} = (-\infty,0] \quad \sqrt{|x|+x} = 4 \rightarrow x+|x| = 16 \rightarrow D_{gof} = \{8\}$$

$$D_{gof} = \mathbb{R} - (-\infty,0] \cup \{8\}$$

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$D_f=[-1,1]$ $\quad D_g=[0,+\infty)$ $\quad \Rightarrow D_{f+g}=[0,1]$

$D_{(f+g)of}=\{x\in[-1,1] : 0\leqslant\underbrace{\sqrt{1-x^2}\leqslant 1}_{\text{همواره برقراره}}\}$ $\Rightarrow$ $\boxed{D_{(f+g)of}=[-1,1]}$

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الف) $\frac{3x+1}{x-2}=t \to 3x+1=tx-2t \to x=\frac{2t+1}{[illegible]} \to f(x)=\frac{8x+4}{[illegible]}+5$

ب) $f(x+\frac{1}{x})=(x+\frac{1}{x})\left((x+\frac{1}{x})^2-3\right) \to f(x)=x^3-3x$

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$g(1)=0 \quad , \quad g(2\sqrt{2})=0$

$g(f(x))=0 \to g(x\sqrt{x})=0$

$$\begin{cases} x\sqrt{x}=1 \to \boxed{x=1} \\ x\sqrt{x}=2 \to \boxed{x=2} \end{cases}$$