

$$f(u) = \begin{cases} \cot \frac{\pi u}{2} & : u \leq 1 \\ \sqrt{u^2 + 1} & : u > 1 \end{cases} \quad f \circ f \left(\frac{1}{2} \right) = 3$$

$$f \left(f \left(\frac{1}{2} \right) \right) = f \left(\sqrt{\frac{1}{4} + 1} \right) = f \left(\sqrt{\frac{5}{4}} \right) = f \left(\frac{\sqrt{5}}{2} \right) = 3$$

(الف) $f(u) = \sqrt{\frac{u-1}{2u}}$ $g(u) = 2 \cos^2 u$ $f \circ g \left(\frac{\pi}{4} \right)$

(ب) $f(u) = [u]$ $g(u) = \frac{u}{1-u}$

$f \left(g \left(\frac{\pi}{4} \right) \right) \rightarrow g \left(2 \cos^2 \frac{\pi}{4} \right) = f(2) = 2$

$f \left(\frac{1}{2} \right) \rightarrow \sqrt{\frac{1/2 - 1}{2 \cdot 1/2}} = \sqrt{\frac{-1/2}{1}} = \sqrt{-1/2}$

$f \left(g \left(\frac{\pi}{4} \right) \right) \rightarrow f(2) \rightarrow f \left(\frac{2}{1-2} \right) \rightarrow f(-2) \rightarrow \left[\frac{-2}{1-(-2)} \right] = \left[\frac{-2}{3} \right] = -1$

$f(u) = \sin u$ $g(u) = u \sqrt{1-u^2}$ $g \circ f \left(\frac{\pi}{2} \right) = 0$

$g \left(f \left(\frac{\pi}{2} \right) \right) \rightarrow g(1) \rightarrow 1 \sqrt{1-1^2} = 0$

$g \left(\frac{\sqrt{2}}{2} \right) = \frac{\sqrt{2}}{2} \sqrt{1 - \left(\frac{\sqrt{2}}{2} \right)^2} = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{1}{2}} = \frac{\sqrt{2}}{2} \sqrt{\frac{1}{2}} = \frac{1}{2}$

$f(u) = \{ (\epsilon, \omega) (\epsilon, \omega) (\lambda, \mu) (\lambda, \mu) \}$ $g(u) = \{ (\epsilon, \epsilon) (\epsilon, \epsilon) (\epsilon, \lambda) (\lambda, \lambda) \}$

الف) $f \circ g \Rightarrow f(g(u)) \rightarrow \epsilon \rightarrow \omega \rightarrow \lambda \rightarrow \mu \rightarrow \epsilon \rightarrow \omega \rightarrow \lambda \rightarrow \mu \rightarrow \epsilon \rightarrow \omega \rightarrow \lambda \rightarrow \mu$

ب) $g \circ f = g(f(u)) \rightarrow \epsilon \rightarrow \omega \rightarrow \lambda \rightarrow \mu \rightarrow \epsilon \rightarrow \omega \rightarrow \lambda \rightarrow \mu \rightarrow \epsilon \rightarrow \omega \rightarrow \lambda \rightarrow \mu$

ج) $f \circ f \Rightarrow f(f(u)) \rightarrow \epsilon \rightarrow \omega \rightarrow \lambda \rightarrow \mu \rightarrow \epsilon \rightarrow \omega \rightarrow \lambda \rightarrow \mu \rightarrow \epsilon \rightarrow \omega \rightarrow \lambda \rightarrow \mu$

د) $g \circ g \Rightarrow g(g(u)) \rightarrow \epsilon \rightarrow \epsilon \rightarrow \lambda \rightarrow \lambda \rightarrow \epsilon \rightarrow \epsilon \rightarrow \lambda \rightarrow \lambda \rightarrow \epsilon \rightarrow \epsilon \rightarrow \lambda \rightarrow \lambda$

$f = \{ (1,1) (1,2) (\epsilon, \omega) (\lambda, \mu) \}$ $g = \{ (1,2) (1,2) (\epsilon, \mu) (\lambda, \mu) \}$

$(1,2) \in f \circ g$ $(1,2) \in g \circ f$

$f \circ g(u) \rightarrow 1 \rightarrow 2 \rightarrow 1 \rightarrow 2 \rightarrow \epsilon \rightarrow \omega \rightarrow \lambda \rightarrow \mu \rightarrow \epsilon \rightarrow \omega \rightarrow \lambda \rightarrow \mu$

$g \circ f(u) \rightarrow 1 \rightarrow 2 \rightarrow 1 \rightarrow 2 \rightarrow \epsilon \rightarrow \omega \rightarrow \lambda \rightarrow \mu \rightarrow \epsilon \rightarrow \omega \rightarrow \lambda \rightarrow \mu$

$$f(f(u)) = \epsilon u + r \rightarrow f(au + s) = a(au + s) + s \Rightarrow a^2 u + as + b = \epsilon u + r$$

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$$g(ru + r) = ru - r$$

$$a(ru + r) + s = ru - r \rightarrow raru + ra + s = ru - r \rightarrow a = 1/0 \quad b = -4/0$$

$$g(f(-1)) = -1$$

$$y = 1/0u - 4/0$$

$$f(u) = \sqrt{u+|u|} \quad g(u) = \frac{1}{ur - \epsilon u}$$

(10)

$$\mathcal{D}_{g \circ f} = \left\{ u \in \mathcal{D}_f, f(u) \in \mathcal{D}_g \right\}$$

$$\sqrt{u+|u|} = 0 \rightarrow u \leq 0$$

$$\sqrt{u+|u|} = \epsilon \rightarrow u = 1$$

$$\mathcal{D}_{g \circ f} = \mathbb{R} - [(-\infty, 0] \cup \{1\}]$$

$$f(u) = \sqrt{1-u^2} \quad g(u) = \sqrt{u}$$

(11)

$$\mathcal{D}_f = [-1, 1] \quad \mathcal{D}_g = [0, +\infty) \Rightarrow \mathcal{D}_{f+g} = \mathcal{D}_f \cap \mathcal{D}_g = [0, 1]$$

$$\mathcal{D}_{(f+g) \circ f} = \left\{ u \in \mathcal{D}_f, f(u) \in \mathcal{D}_{f+g} \right\} \Rightarrow \left\{ u \in [-1, 1], -\epsilon \sqrt{1-u^2} \leq 1 \right\}$$

$$\frac{-\epsilon \sqrt{1-u^2}}{2\epsilon} \leq 1 \rightarrow 1 \geq -u^2 + 1 \rightarrow -u^2 \leq 0 \rightarrow u = 0$$

$$\mathcal{D}_{f+g} \circ f = [-1, 1]$$

$$f\left(\frac{ru+1}{u-r}\right) = \epsilon u + \delta$$

(12)

$$\frac{ru+1}{u-r} = \epsilon \rightarrow u = -\frac{-r\epsilon-1}{r\epsilon-r} \rightarrow \frac{r\epsilon+1}{\epsilon-r} = u$$

$$f(\epsilon) = \epsilon \left(\frac{r\epsilon+1}{\epsilon-r} \right) + \delta \rightarrow f(u) = \frac{\epsilon u + \epsilon}{\epsilon - r} + \delta$$

$$f(u + \frac{1}{u}) = u^r + \frac{1}{u^r}$$

$$(u + \frac{1}{u})^r = \epsilon^r = u^r + \frac{1}{u^r} + ru + \frac{r}{u}$$

$$\left(\frac{u + \frac{1}{u}}{\epsilon} \right)^r = \epsilon^r (u + \frac{1}{u}) \Rightarrow f(\epsilon) = \epsilon^r - r\epsilon$$

$$g \circ f(u) \rightarrow g(f(u)) \rightarrow g(u\sqrt{u}) = 0$$

$$u\sqrt{u} = r\sqrt{r} \rightarrow u = r$$

$$u\sqrt{u} = 1 \rightarrow u = 1 \Rightarrow \boxed{u = 1}$$

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