

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & : x \leq 1 \\ \sqrt{x^2 + 1} & : x > 1 \end{cases}$$

$$f\left(\frac{1}{4}\right) = \cot \frac{\pi}{4} = \cot \frac{\pi}{4} = \sqrt{4} \quad f(\sqrt{4}) = \sqrt{(\sqrt{4})^2 + 1} = \sqrt{5}$$

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الف) $g(x) = \pi \cos^2 x$ $f\left(g\left(\frac{\pi}{4}\right)\right)$ $1 \cos^2\left(\frac{\pi}{4}\right) = 1 \times \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$f\left(\frac{1}{4}\right) = \sqrt{\frac{1}{16} + 1} = \sqrt{\frac{17}{16}} = \frac{\sqrt{17}}{4}$

$f(x) = \sqrt{\frac{x^2 + 1}{x^2}} = \sqrt{\frac{1}{x}}$

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ب) $g(x) = \frac{x}{1-x}$ $f(g(\sqrt{x}))$ $\frac{\sqrt{x}}{1-\sqrt{x}} \times \left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right) = -\sqrt{x} - x$

$f(x) = [x]$ $f(-\sqrt{x} - x) = [-\sqrt{x} - x] = -x$

$g(x) = x \sqrt{1-x^2}$ $g\left(f\left(\frac{\pi}{4}\right)\right)$ $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$f(x) = \sin x$ $g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{1}{2}$

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$g(x) = \left\{ \left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right) \right\}$ $f(x) = \left\{ \left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right) \right\}$

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الف) $f(g(x)) \Rightarrow \left\{ \left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right), \left(\frac{1}{2}, \frac{1}{2} \right) \right\}$

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$f = \left\{ (1, 1), (2, 2), (3, 3), (4, 4) \right\}$ $g = \left\{ (1, 2), (2, 3), (3, 4), (4, 5) \right\}$ $(a, b) = (1, 2)$

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$f(g(x)) = (1, 1)(2, 2)(3, 3)(4, 4) = (1, 1)(a, 2) \Rightarrow a = 1$

$g(f(x)) = (2, 2)(3, 3)(4, 4)(5, 5) = (2, 2)(1, ?) = (2, 2)(1, ?) \Rightarrow ? = 2 \Rightarrow b = 2$

$f(f(x)) = 4x + 3$ $g(2x + 3) = 2x + 3$ $g(f(-1)) =$ $f(x) = 4x + 3 \rightarrow x = -1 \rightarrow -1$

$f(x) = ax + b$

$f(f(x)) = a(ax + b) + b$

$ax + b = -1 \Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2}$ $g(-1) = 2 \times (-1) - 2 = -4$

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نورالدين

$$f(x) = \sqrt{x+1} \quad , \quad g(x) = \frac{1}{x^2 - 5}$$

$$\sqrt{x+1} \Rightarrow x+1 \geq 0$$

$$|x| \geq -x \rightarrow \mathbb{R} \text{ لكل } x$$

1 ✓

$$D_{g \circ f} \Rightarrow g(f(x))$$

$$x^2 - 5x = 0$$

$$x(x-5) = 0 \rightarrow x = 5 \Rightarrow x \neq 5$$

ممنوع

$$\sqrt{x+1} \neq 0 \rightarrow x \neq -1$$

$$\sqrt{x+1} \neq 5 \rightarrow x \neq 24$$

$$D_{g \circ f} = (0, +\infty) - \{1\}$$

$$D_{g \circ f} = \mathbb{R} - \{5\}$$

$$g(x) = \sqrt{x} \quad , \quad f(x) = \sqrt{1-x^2}$$

$$1-x^2 \geq 0 \Rightarrow 1 \geq x^2 \rightarrow D_f = [-1, 1]$$

$$\sqrt{x} \Rightarrow x \geq 0$$

$$D_{f \circ g} = D_f \cap D_g = \textcircled{1} \cap \textcircled{2} = [0, 1]$$

2 ✓

$$f(g(x)) = \sqrt{\frac{x}{1-x^2}} \Rightarrow \sqrt{\frac{x}{1-x^2}} \leq 1 \Rightarrow 1-x^2 \leq 1 \Rightarrow -x^2 \leq 0 \Rightarrow \mathbb{R}$$

$$\Rightarrow D_{f \circ g} = [-1, 1]$$

$$f\left(\frac{x+1}{x-1}\right) = 5x+2$$

$$\frac{x+1}{x-1} = t \Rightarrow x+1 = tx-t \Rightarrow 1+xt = tx-t \Rightarrow x = \frac{1+xt}{t-x}$$

$$f(t) = 5\left(\frac{1+xt}{t-x}\right) + 2 \Rightarrow f(x) = 5\left(\frac{1+xt}{x-1}\right) + 2$$

3 ✓

$$f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3}$$

$$x + \frac{1}{x} = t \Rightarrow t^3 = \left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x + \frac{3}{x} \Rightarrow x^3 + \frac{1}{x^3} = t^3 - 3\left(x + \frac{1}{x}\right)$$

$$f(t) = t^3 - 3t \Rightarrow f(x) = x^3 - 3x$$

$$t^3 - 3\left(x + \frac{1}{x}\right)$$

$$\left. \begin{array}{l} g(1) = 0 \\ g(\sqrt{x}) = 0 \end{array} \right\} \begin{array}{l} \text{نقطة تقاطع } g(x) \text{ مع المحور } x \\ y = 0 \end{array}$$

$$g \circ f(x) = g(f(x)) \rightarrow f(x) = 1 \Rightarrow x\sqrt{x} = 1 \Rightarrow x = 1$$

$$f(x) = \sqrt{x} \Rightarrow x\sqrt{x} = \sqrt{x} \Rightarrow x = 1$$

4 ✓