

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & x < 1 \\ \sqrt{x^2 + 1} & x > 1 \end{cases} \quad f\left(\frac{1}{2}\right) = \cot \frac{\pi \cdot \frac{1}{2}}{4} = \cot \frac{\pi}{8} \rightarrow \cot \frac{\pi}{8} \rightarrow \sqrt{3}$$

$$\rightarrow f\left(\sqrt{3}\right) = \sqrt{(\sqrt{3})^2 + 1} = 2$$

$$f \circ f\left(\frac{1}{2}\right) = f\left(f\left(\frac{1}{2}\right)\right) = \underline{\underline{2}}$$

$$f(x) = [x] \rightarrow [-0.5] = -1 \quad (ب)$$

$$f\left(\frac{1}{2}\right) = \sqrt{\frac{1}{2} - 1} = \sqrt{\frac{-1}{2}}$$

$$g(x) = \frac{x}{1-x} \rightarrow \frac{\sqrt{2}}{1-\sqrt{2}} \cdot \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -2-\sqrt{2}$$

$$g(x) = 2 \cos^2 x \rightarrow 2 \cos^2 \frac{\pi}{4} = \frac{1}{2}$$

$$f \circ g(\sqrt{2}) = -1$$

$$f \circ g\left(\frac{\pi}{4}\right) = \underline{\underline{\frac{\sqrt{2}}{2}}}$$

$$f(x) = \sin x \rightarrow \sin\left(\frac{\pi}{2}\right) = \frac{\sqrt{2}}{2}$$

$$g(x) = x\sqrt{1-x^2} \rightarrow \frac{\sqrt{2}}{2} \sqrt{1-\frac{2}{4}} = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} = \frac{1}{2}$$

$$g \circ f\left(\frac{\pi}{2}\right) = \underline{\underline{\frac{1}{2}}}$$

$$\begin{aligned} \varepsilon &\rightarrow \delta \rightarrow \Lambda \\ \gamma &\rightarrow \delta \rightarrow \delta \\ \varepsilon &\rightarrow \Lambda \rightarrow \Lambda \\ \Lambda &\rightarrow \Lambda \rightarrow X \end{aligned}$$

$$g \circ g(x)$$

$$f \circ f(x)$$

$$g \circ f(x)$$

$$f(g(x)), f \circ g(x)$$

$$\begin{aligned} \varepsilon &\rightarrow \delta \rightarrow X / \Lambda \rightarrow \Lambda \rightarrow X \\ \delta &\rightarrow \Lambda \rightarrow X / \Lambda \rightarrow \Lambda \rightarrow X \end{aligned}$$

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$$g \circ g(x) = \{(\varepsilon, \Lambda) (\delta, \delta) (\gamma, \Lambda) \}$$

$$f \circ f = \emptyset$$

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$$f \circ g(x) = \{ (\varepsilon, \delta) (\delta, \delta) (\gamma, \Lambda) (\Lambda, \Lambda) \}$$

$$f = \{ (\varepsilon, 1) (\delta, 2) (\gamma, 5) (1, 0) \}$$

$$g(x) = \{ (1, 2) (2, 1) (\alpha, \varepsilon) (b, 1) \}$$

$$(\varepsilon, 2) \in f \circ g$$

$$(\varepsilon, 1) \in g \circ f$$

$$f \circ g \rightarrow \begin{cases} 1 \rightarrow 2 \rightarrow 1 \\ 2 \rightarrow 1 \rightarrow 0 \\ \alpha \rightarrow \varepsilon \rightarrow 2 \\ b \rightarrow 1 \rightarrow 0 \end{cases} \Rightarrow a = f$$

$$(\varepsilon, 5)$$

$$g \circ f \rightarrow \begin{cases} 1 \rightarrow 1 \rightarrow 1 \\ 2 \rightarrow 2 \rightarrow \alpha \\ \varepsilon \rightarrow 5 \rightarrow 1 \\ 1 \rightarrow 0 \rightarrow \alpha \end{cases} \Rightarrow b = 5$$

$$f(F(x)) = 5x + 1 \rightarrow a(ax+b) + b = 5x + 1 \rightarrow a^2x + ab + b = 5x + 1$$

$$g(2x+1) = 2x-1 \rightarrow a(2x+1) + b = 2x-1 \rightarrow 2ax + a+b = 2x-1 \rightarrow a = \frac{2}{2} = 1, b = -1-2 = -3$$

$$g \circ f(-1) = -1 \Rightarrow f = 2x+1, g = \frac{2}{2}x - \frac{1}{2}$$

$$f(-1) = -2+1 = -1, g(-1) = -\frac{2}{2} + \frac{1}{2} = -\frac{1}{2} = -1$$

$$f(x) = \sqrt{x+|x|}, g(x) = \frac{1}{x^2-5x}$$

$$D_{g \circ f} = \left\{ x \in D_f : f(x) \in D_g \right\} = \left\{ x \in \mathbb{R} : \sqrt{x+|x|} \in \mathbb{R} - \{0, 5\} \right\} \Rightarrow D_{g \circ f} = (0, \infty) - \{1\}$$

$$D_f = x+|x| \geq 0 \rightarrow \mathbb{R}$$

$$D_g = x^2 - 5x \neq 0 \rightarrow x(x-5) \neq 0 \rightarrow x \neq 0, x \neq 5$$

$$f(x) = \sqrt{1-x^2}, g(x) = \sqrt{x}$$

$$D_{(f+g) \circ f} = \left\{ x \in D_f : f(x) \in D_{f+g} \right\} = \left\{ x \in [-1, 1] : x \in [-1, 1] \right\} \Rightarrow D = [-1, 1]$$

$$D_{f+g} = D_f \cap D_g \Rightarrow [-1, 1] \cap [0, \infty) = [0, 1]$$

$$D_g = x \geq 0, D_f = [-1, 1] \Rightarrow 0 \leq 1-x^2 \leq 1 \Rightarrow |x| \leq 1 \Rightarrow x \in [-1, 1]$$

$$الف) f\left(\frac{3x+1}{x-2}\right) = 5x+3$$

$$\frac{3x+1}{x-2} = t \rightarrow x = \frac{-t+1}{t-3} = \frac{1-t}{t-3}$$

$$\rightarrow f(t) = f\left(\frac{1-t}{t-3}\right) = 5x+3 = \frac{5(1-t)}{t-3} + 3$$

$$\rightarrow f(x) = \frac{5x+8}{x-3} + 3$$

$$ب) f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$$

$$x + \frac{1}{x} = t \rightarrow \left(x + \frac{1}{x}\right)^2 = t^2 \rightarrow x^2 + \frac{1}{x^2} = t^2 - 2$$

$$f(t) = t^2 - 2$$

$$f(x) = x^2 - 2x$$

$$f(x) = x\sqrt{x}$$

$$g \circ f(x) \rightarrow g(f(x)) = 0 \rightarrow f(x) = 1 \rightarrow x\sqrt{x} = 1 \rightarrow x = 1$$

$$g(1) = 0, g(2\sqrt{2}) = 0 \rightarrow f(x) = 2\sqrt{2} = x\sqrt{x} = 2\sqrt{2} \rightarrow x = 2$$

$$\text{اختلاف طول مقاطين} = x_2 - x_1 = 2 - 1 = 1$$