$gog(x)$: $4 \to 6 \to 8$, $2 \to 4 \to 6$, $6 \to 8 \to 10$, $8 \to 10$; $gog(x) = \{(4,8)(6,10)(2,6)\}$

(5)

$fog(x)$: $1 \to 2 \to 1$, $3 \to 1 \to 7$, $a \to 3 \to 2$, $b \to 1 \to 7$; $fog(x)$: $(1,1)(3,7)(a,2)(b,1)$

$(a,b) = (4,1)$

$gof(x)$: $2 \to 1 \to 2$, $3 \to 2 \to 3$, $4 \to 5 \to (1) \leftarrow = (b)$, $1 \to 7$

(6)

$f(x) = ax+b \quad f(f(x)) \Rightarrow$

$f(ax+b) = 4x+3 \to 4(ax+b)+3$

$\underbrace{4ax}_{4x} + \underbrace{4b + 3}_{3} = 4x+3 \to \boxed{a=1}\ \boxed{b=0} \Rightarrow f(x) = x$

$g(\underbrace{2x+3}_{t}) = 3x-2 \quad 2x+3 = t \to x = \frac{t-3}{2}$

$g(x) = \frac{3t-9}{2} - 2 = \frac{3t-9-4}{2} = \frac{3t-13}{2}$

$g(f(x)) \xrightarrow{x=-1} \frac{-3-13}{2} = \boxed{-8}$

(7)

$\sqrt{\ldots} \quad \ldots \quad \sqrt{\frac{\ldots}{x}} = \sqrt{8}$

$D = \mathbb{R} - [\ldots] \cup \{\sqrt{8}\}$

$x^2 - 4x = 0$

$x = 0, 4$

(8)

$D_f = [-1,1]$, $D_g = [0,+\infty) \Rightarrow D_{f+g} = [0,1]$

$D_{(f+g)\circ f} = \{x \in [-1,1] \mid 0 \leq \sqrt{1-x^2} \leq 1\}$

$0 \leq \sqrt{1-x^2} \leq 1 \to 1-x^2 \leq 1 \to -x^2 \leq 0$

همیشه برقرار است. انعکاسی ... همیشه برقرار.

$D_{(f+g)\circ f} = [-1,1]$

(9)

الف) $\frac{3x+1}{x-2} = t \to x = \frac{2t+1}{t-3}$

$f(x) = 4\left(\frac{2t+1}{t-3}\right) + 5 \to \frac{8t+4}{t-3} + \frac{5}{1} = \frac{8t+4+5t-15}{t-3} = \frac{13t-11}{t-3}$

$f(x) = \frac{13x-11}{x-3}$

ب) $x + \frac{1}{x} = t \quad f(x+\frac{1}{x}) = (x+\frac{1}{x})^3 - 3(x+\frac{1}{x})$

$f(t) = t^3 - 3t \to f(x) = x^3 - 3x$

(10)

$f(x) = 1 \to x\sqrt{x} = 1 \to \boxed{x=1}$

$f(x) = 2\sqrt{2} \to x\sqrt{x} = 2\sqrt{2} \quad \boxed{x=2}$

اختلاف: $2-1=1$

$\sqrt{3} = 1/7$ نازنین داوود .

(۱)

$f(f(\frac{1}{4})) = \sqrt{(\sqrt{3})^2 + 1} = \sqrt{4} = 2$

$f(\frac{1}{4}) = \cot \frac{\frac{1}{4}\pi}{4 \quad \frac{1}{4}\pi} = \sqrt{3}$

30°

(۲)

الف) $f(g(\frac{\pi}{3}))$ $\quad 2\cos^2(60^\circ) = \frac{1}{2}$

$f(\frac{1}{2}) = \sqrt{\frac{2(2)-1}{4}} = \frac{\sqrt{3}}{2}$

ب) $f(g(x)) = [\frac{x}{1-x}] \xrightarrow{x=\sqrt{2}} [\frac{\sqrt{2}}{1-\sqrt{2}}] \longrightarrow$ $1/4$

$[\frac{\sqrt{2}}{1-\sqrt{2}} \times (\frac{1+\sqrt{2}}{1+\sqrt{2}})] = [\frac{\sqrt{2}+2}{\underbrace{1-2}_{-1}}] = [\underbrace{-2-\sqrt{2}}_{-3/4\ \text{حدوداً}}] = -4$

(۳)

$gof(x) = g(f(x)) = \sin x \sqrt{\underbrace{1-\sin^2 x}_{\cos^2 x}} =$

$\sin x |\cos x| \xrightarrow{x=\frac{\pi}{4}} \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$

(۴)

$fog(x) =$ $g(x)$:
$4 \to 6 \to 5$
$2 \to 4 \to 5$
$6 \to 8 \to 12$
$8 \to 10 \to 12$

$fog(x) = \{(4,5)\ (2,5)\ (6,12)\ (8,12)\}$

$gof(x) =$
$4 \to 5$
$6 \to 5$
$8 \to 12$
$10 \to 12$
$\varnothing$

$fof(x) =$
$4 \to 5$
$6 \to 5$
$8 \to 12$
$10 \to 12$
$\varnothing$