۱۹,۲۵

نام و نام خانوادگی ......کیانا عباسی...... پاسخنامه تشریحی تکلیف شماره ..۹.. کلاس ......11A......

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$f(\frac{2}{3}) = \cot\frac{\frac{2}{3}\pi}{\frac{4}{1}} \Rightarrow \cot\frac{\pi}{6} = \sqrt{3} \Rightarrow f(\sqrt{3}) = \sqrt{(\sqrt{3})^2+1} = \sqrt{4} = 2$

$\sqrt{4} = 2$ ✓

$\sqrt{4} = -2$ ✗

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الف) $g(\frac{\pi}{3}) = 2\cos^2\frac{\pi}{3} = \frac{1}{2} \Rightarrow f(\frac{1}{2}) = \sqrt{\frac{4-1}{4}} = \frac{\pm\sqrt{3}}{2}$ ✓

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ب) $g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \Rightarrow f(\frac{\sqrt{2}}{1-\sqrt{2}}) = [\frac{\sqrt{2}}{1-\sqrt{2}}] = -4$

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$f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2} \times \underbrace{\sqrt{1-\frac{2}{4}}}_{\frac{2}{4}} \Rightarrow \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$

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الف) $fog(x) = \{(4,5),(2,5),(9,12)(8,12)\}$

ب) $gof(x) = \emptyset$

ج) $fof(x) = \emptyset$

د) $gog(x) = \{(4,8)(2,9)(7,10)\}$

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$f(g(x)) = \{(1,1)(3,7)(a,2)(b,7)\} \xrightarrow{(4,2)\in fog} a=4$

$g(f(x)) = \{(3,2)\underbrace{(4,1)}\}$

$g(4) = 1 \Rightarrow b = 5$

$\Rightarrow (a,b) = (4,5)$

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$f(f(x)) = f(ax+b) = a(ax+b)+b = a^2x+ab+b = 4x+3 \Rightarrow a^2=4,\ ab+b=3 \ \begin{cases} a=2 \\ b=1 \end{cases} \to y=2x+1$

$g(f(-1)) \Rightarrow f(-1) = -2+1 = -1 \to g(-1) \Longrightarrow 2x+3=-1 \to \underline{x=-2} \Rightarrow g(-1) = -6-2 = -8$

(۲)

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$① \ x+|x| \geq 0 \to R \quad ② \ x \geq 0 \to (0,+\infty) \to ① \cap ② = (0,+\infty) \to D_f$

$x^2-4x \neq 0 \to \Delta = 16 \to \frac{4 \pm 4}{2} \to 4,\ 0 \to R-\{0,4\} \to D_g$

$D_{gof} = \{x \in D_f,\ f(x) \in D_g\}$

$D_{gof} = \{x \in (0,+\infty),\ \underbrace{\sqrt{x+|x|}}_{x \neq 8} \in R-\{0,4\}\} \to D_{gof} = (0,+\infty)-\{8\}$

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$D_{(f+g)of} = \{x \in D_f,\ f(x) \in D_{f+g}\} \to D_f = 1-x^2 \geq 0 \to -1 \leq x \leq 1$

$D_g = x \geq 0 \to [0,+\infty) \Rightarrow D_{f+g} = D_f \cap D_g = [0,1]$

$\to D_{(f+g)of} = \{x \in [-1,1],\ \sqrt{1-x^2} \in [0,1]\} \Rightarrow D_{(f+g)of} = [-1,1] \cap [0,1] = \cancel{[0,1]}\ [-1,1]$

$x^2 \geq 0 \qquad 0 \leq \sqrt{1-x^2} \leq 1 \to \leq x \leq 1$

(۱٫۷۵)

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الف) $\frac{3x+1}{x-2} = t \to tx-2t = 3x+1 \Rightarrow x(t-3) = -2t-1 \Rightarrow x = \frac{2t+1}{\cancel{-t+3}\ t-3} \Rightarrow f(t) = 4\left(\frac{-2t-1}{-t+3}\right)+5$

$\Rightarrow f(x) = \cancel{\frac{-8x-4}{-x+3}+5} \qquad \frac{13x-11}{x-3}$

(۱٫۵)

ب) $x+\frac{1}{x} = t \to \left(x+\frac{1}{x}\right)^3 \to x^3+\frac{1}{x^3}+3x+\frac{3}{x} \to x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$\Rightarrow f(t) = t^3-3t \Rightarrow f(x) = x^3-3x$

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$g(f(x)) = 0 \to g(?) = 0 \to g(1) = 0 \to f(x) = 1 \to x\sqrt{x} = 1 \Rightarrow x = 1$

$g(2\sqrt{2}) = 0 \to f(x) = 2\sqrt{2} \to x\sqrt{x} = 2\sqrt{2} \to x = 2$

$2-1 = 1$

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