نام و نام خانوادگی ......لیلا عباسی...... پاسخنامه تشریحی تکلیف شماره ..۹.... کلاس .....11A......

۱	$f(\frac{2}{3}) = \cot \frac{\frac{2}{3}\pi}{\frac{4}{1}} \Rightarrow \cot\frac{\pi}{6} = \sqrt{3} \Rightarrow f(\sqrt{3}) = \sqrt{(\sqrt{3})^2+1} = \sqrt{4} = 2$
۲	الف) $g(\frac{\pi}{3}) = 2\cos^2\frac{\pi}{3} = \frac{1}{2} \Rightarrow f(\frac{1}{2}) = \sqrt{\frac{4-1}{4}} = \pm\frac{\sqrt{3}}{2}$ ب) $g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \Rightarrow f(\frac{\sqrt{2}}{1-\sqrt{2}}) = [\frac{\sqrt{2}}{1-\sqrt{2}}] = -4$
۳	$f(\frac{\pi}{4}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2} \times \sqrt{\underbrace{1-\frac{2}{4}}_{\frac{2}{4}}} \Rightarrow \frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$
۴	الف) $fog(x) = \{(4,5),(2,5),(9,12)(8,12)\}$ ب) $gof(x) = \emptyset$ ج) $fof(x) = \emptyset$ د) $gog(x) = \{(4,8)(2,9)(7,10)\}$
۵	$f(g(x)) = \{(1,1)(3,7)(a,2)(b,7)\} \xrightarrow{(4,2)\in fog} a=4$ $g(f(x)) = \{(2,2)\underbrace{(4,1)}\}$ $\Rightarrow (a,b)=(4,5)$ $g(4)=1 \Rightarrow b=5$

۶

$f(f(x)) = f(ax+b) = a(ax+b)+b = a^2x+ab+b = 4x+3 \Rightarrow a^2=4,\ ab+b=3 \begin{cases} a=2 \\ b=1 \end{cases} \rightarrow y=2x+1$

$g(f(-1)) \Rightarrow f(-1) = -2+1 = -1 \rightarrow g(-1) \Rightarrow 2x+3=-1 \rightarrow x=-2 \Rightarrow g(-1) = -6-2 = -8$

۷

$① x+|x| \geq 0 \rightarrow R \quad ② x \geq 0 \rightarrow (0,+\infty) \rightarrow ① \cap ② = (0,+\infty) \rightarrow D_f$

$x^2-4x \neq 0 \rightarrow \Delta = 16 \rightarrow \frac{4\pm4}{2} \rightarrow 4,\ 0 \rightarrow R-\{0,4\} \rightarrow D_g$

$D_{gof} = \{x \in D_f,\ f(x) \in D_g\}$

$D_{gof} = \{x \in (0,+\infty),\ \underbrace{\sqrt{x+|x|}}_{x \neq 8} \in R-\{0,4\}\} \rightarrow D_{gof} = (0,+\infty)-\{8\}$

۸

$D_{(f+g)of} = \{x \in D_f,\ f(x) \in D_{(f+g)}\} \Rightarrow D_f = 1-x^2 \geq 0 \rightarrow -1 \leq x \leq 1$

$D_g = x \geq 0 \rightarrow [0,+\infty) \Rightarrow D_{f+g} = D_f \cap D_g = [0,1]$

$\rightarrow D_{(f+g)of} = \{x \in [-1,1],\ \sqrt{1-x^2} \in [0,1]\} \Rightarrow D_{(f+g)of} = [-1,1] \cap [0,1] = [0,1]$

$0 \leq \sqrt{1-x^2} \leq 1 \rightarrow 0 \leq x \leq 1$

۹

الف) $\frac{3x+1}{x-2} = t \rightarrow tx-2t = 3x+1 \Rightarrow x(t-3) = -2t-1 \Rightarrow x = \frac{-2t-1}{-t+3} \Rightarrow f(t) = \frac{4(-2t-1)}{-t+3}+5$

$\Rightarrow f(x) = \frac{-8x-4}{-x+3}+5$

ب) $x+\frac{1}{x} = t \rightarrow (x+\frac{1}{x})^3 \rightarrow x^3+\frac{1}{x^3}+3x+\frac{3}{x} \rightarrow x^3+\frac{1}{x^3} = (x+\frac{1}{x})^3 - 3(x+\frac{1}{x})$

$\Rightarrow f(t) = t^3-3t \Rightarrow f(x) = x^3-3x$

۱۰

$1 ?\ \ 2\sqrt{2} ?$

$g(f(x)) = 0 \rightarrow g(?) = 0 \rightarrow g(1) = 0 \rightarrow f(x) = 1 \rightarrow x\sqrt{x} = 1 \Rightarrow x=1$

$g(2\sqrt{2}) = 0 \rightarrow f(x) = 2\sqrt{2} \rightarrow x\sqrt{x} = 2\sqrt{2} \rightarrow x=2$

$2-1 = 1$