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نام و نام خانوادگی ......[illegible]...... پاسخنامه تشریحی تکلیف شماره ...۹... کلاس ...[illegible]...

۱	$f(x)=\begin{cases}\cot\frac{\pi x}{4} & ; x\le 1\\ \sqrt{x^2+1} & ; x>1\end{cases}$ $\quad f\circ f\left(\frac{2}{3}\right)=f\left(f\left(\frac{2}{3}\right)\right)$ ۳ $f\left(\frac{2}{3}\right)=\cot\frac{\pi}{6}=\cot 30^\circ=\sqrt{3} \Rightarrow f(\sqrt{3})=\sqrt{(\sqrt{3})^2+1}=\sqrt{4}=2$
۲	الف) ۲ $f\left(\frac{1}{x}\right)=\sqrt{\frac{2x-1}{x^2}} \quad g(x)=2\cos^2 x \quad f\circ g\left(\frac{\pi}{3}\right)$ $g\left(\frac{\pi}{3}\right)=2\cos^2 60^\circ=\frac{1}{2} \Rightarrow if\ x=2 \Rightarrow f\left(\frac{1}{2}\right)=\sqrt{\frac{2(2)-1}{4}}=\frac{\sqrt{3}}{2}$ ب) $f(x)=[x] \quad g(x)=\frac{x}{1-x} \quad f\circ g(\sqrt{2})$ $g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}=\frac{\sqrt{2}+2}{1-2}=-2-\sqrt{2} \Rightarrow f(-2-\sqrt{2})=[-2-\sqrt{2}]$ $\Rightarrow [-\sqrt{2}]-2=-2-2=-4$
۳	۳ $f(x)=\sin x \quad g(x)=x\sqrt{1-x^2} \quad g\circ f\left(\frac{\pi}{4}\right)$ $f\left(\frac{\pi}{4}\right)=\sin 45^\circ=\frac{\sqrt{2}}{2} \Rightarrow g\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{2}\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}=\frac{\sqrt{2}}{2}\sqrt{\frac{2}{4}}=\frac{2}{4}$ $=\frac{1}{2}$
۴	$f(x)=\{(4,5),(6,5),(8,12),(10,12)\} \quad g(x)=\{(4,6),(2,4),(6,8),(8,10)\}$ الف) $f\circ g(x)=\{(4,5),(2,5),(6,12),(8,12)\}$ ۳ ب) $g\circ f(x)=\varnothing$ ج) $f\circ f(x)=\varnothing$ د) $g\circ g(x)=\{(4,8),(2,6),(6,10)\}$
۵	$f=\{(2,1),(3,2),(4,5),(1,7)\} \quad g=\{(1,2),(3,1),(a,3),(b,1)\}$ $(4,2)\in f\circ g \quad (4,1)\in g\circ f \quad g(\underbrace{f(4)}_{5})=1 \Rightarrow b=5$ ۲ $(a,b)=? \Rightarrow (4,5)$ $f(g(4))=2 \Rightarrow g(4)=3 \Rightarrow a=4$

$2x+3=-1 \Rightarrow 2x=-4 \Rightarrow x=-2$

$f(f(x)) = 4x+3 \qquad g(2x+3) = 3x-2$

$gof(-1) = ? \Rightarrow \begin{cases} f(-1) = 2(-1)+1 = -1 \Rightarrow g(-1) = \underbrace{3(-2)-2}_{-6} = -8 \\ f(-1) = -2(-1)-3 = -1 \end{cases}$

$f(x) = ax+b \Rightarrow fof = a(ax+b)+b = a^2x+ab+b = 4x+3$ ۶

$\Rightarrow a^2=4 \Rightarrow a=\pm 2 \quad , \quad ab+b=3 \begin{cases} a=2 \Rightarrow 3b=3 \Rightarrow b=1 \\ a=-2 \Rightarrow -b=3 \Rightarrow b=-3 \end{cases}$

$f(x) = \sqrt{x+|x|} \qquad g(x) = \dfrac{1}{x^2-4x} \qquad D_f \Rightarrow x+|x| \geq 0$

$a+|a| \geq 0 \Rightarrow D_f = \mathbb{R}$

$D_{gof} = (0,+\infty) - \{8\} \qquad * f(x) \in D_g$ ۷

$\Updownarrow \qquad \Updownarrow\ f(x) \neq 0,4 \qquad D_g \Rightarrow x^2-4x \neq 0$

$\{x \in D_f \mid f(x) \in D_g\}^{*} \qquad \Updownarrow \qquad \Updownarrow\ x(x-4) \neq 0$

$x \neq \mathbb{R}^-, 0, 8 \qquad x \neq 0 \quad x \neq 4$

$x \neq 8 \Leftarrow x+|x| \neq 16 \Leftarrow \sqrt{x+|x|} \neq 4 \qquad D_g = \mathbb{R} - \{0,4\}$

$f(x) = \sqrt{1-x^2} \qquad g(x) = \sqrt{x} \quad R_f \subseteq [-1,1]$

$D_f = 1-x^2 \geq 0$

$D(f+g)of = [-1,1] \Leftarrow \quad \Uparrow \quad \Updownarrow$

$D_{(f+g)of} = \{x \in D_f \mid f(x) \in D_{f+g}\} \qquad R_f = [0,1] \qquad 1 \geq x^2$

$\Uparrow \qquad -1 \leq x \leq 1 \Rightarrow D_f = [-1,1]$ ۸

$R_f \Rightarrow y = 1-x^2 \Rightarrow x^2 = 1-y \Rightarrow 1-y \geq 0 \Rightarrow y \leq 1$

$D_g = x \geq 0 \Rightarrow [0,+\infty)$

$D_{f+g} = D_f \cap D_g = [-1,1] \cap [0,+\infty) = [-1,1] \Rightarrow R_f \subseteq [-1,1]$

الف) $f\left(\dfrac{3x+1}{x-2}\right) = 4x+5$

$\dfrac{3x+1}{x-2} = t \Rightarrow tx-2t = 3x+1 \Rightarrow tx-3x = 1+2t \Rightarrow x(t-3) = 1+2t$

$\Rightarrow x = \dfrac{1+2t}{t-3} \Rightarrow f(t) = 4\left(\dfrac{1+2t}{t-3}\right)+5 = \dfrac{4+8t+5t-15}{t-3} = \dfrac{13t-11}{t-3} \Rightarrow f(x) = \dfrac{13x-11}{x-3}$ ۹

ب) $f\left(x+\dfrac{1}{x}\right) = x^3+\dfrac{1}{x^3} \Rightarrow f(t) = t^3-3(t) \Rightarrow f(x) = x^3-3x$

$x+\dfrac{1}{x} = t \Rightarrow \left(x+\dfrac{1}{x}\right)^3 = x^3+\dfrac{1}{x^3}+3x+\dfrac{3}{x} \Rightarrow x^3+\dfrac{1}{x^3} = \left(x+\dfrac{1}{x}\right)^3 - 3\left(x+\dfrac{1}{x}\right)$

$f(x) = x\sqrt{x} \qquad g(f(x))$

$\text{if } x = 1, 2\sqrt{2} \Rightarrow g(x) = 0$

$\Updownarrow$ ۱۰

$f(x) = 1 \Rightarrow x\sqrt{x} = 1 \Rightarrow x = 1 \qquad \Rightarrow 2-1 = 1$

$f(x) = 2\sqrt{2} \Rightarrow x\sqrt{x} = 2\sqrt{2} \Rightarrow x = 2$