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$$f(x) = \begin{cases} \cot \frac{\pi x}{\varepsilon} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases} \Rightarrow \cot \frac{\frac{\sqrt{3}}{2} \pi}{\frac{1}{2}} \Rightarrow \cot \frac{\pi}{\varepsilon} \Rightarrow \frac{\sqrt{3}}{\frac{1}{2}} \quad (1)$$

$$\Rightarrow \sqrt{(\sqrt{3})^2 + 1} \Rightarrow \sqrt{4} = 2 \Rightarrow \sqrt{3}$$

$$f\left(\frac{f\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}\right) = 2$$

الف) $f\left(g\left(\frac{\sqrt{3}}{2}\right)\right) \Rightarrow$

$$g(x) = \varepsilon \cos \frac{\pi}{\varepsilon} \Rightarrow \varepsilon \left(\frac{1}{\varepsilon}\right)^2 \Rightarrow \frac{1}{\varepsilon}$$

$$f\left(\frac{1}{\varepsilon}\right) = \sqrt{\frac{\varepsilon^2 - 1}{\varepsilon^2}} \Rightarrow \frac{1}{\varepsilon} = \frac{1}{\varepsilon} \Rightarrow \varepsilon = 2 \Rightarrow \sqrt{\frac{2 \times 2 - 1}{2^2}} \Rightarrow \frac{\sqrt{3}}{2}$$

ب) $f(g(\sqrt{2}))$

$$f(x) = [x] \Rightarrow \left[\frac{\sqrt{2}}{1 - \sqrt{2}} \right] = -1$$

$$g(x) = \frac{x}{1-x} \Rightarrow \frac{\sqrt{2}}{1-\sqrt{2}}$$

$$g\left(f\left(\frac{\pi}{\varepsilon}\right)\right) \quad f\left(\frac{\pi}{\varepsilon}\right) = \sin \frac{\pi}{\varepsilon} \Rightarrow \frac{\sqrt{2}}{\varepsilon} \quad (2)$$

$$g\left(\frac{\sqrt{2}}{\varepsilon}\right) = \frac{\sqrt{2}}{\varepsilon} \sqrt{1 - \left(\frac{\sqrt{2}}{\varepsilon}\right)^2} \Rightarrow \frac{\sqrt{2}}{\varepsilon} \sqrt{1 - \frac{1}{\varepsilon^2}} \Rightarrow \frac{\sqrt{2}}{\varepsilon} \times \frac{1}{\varepsilon} = \frac{1}{\varepsilon}$$

الف) $f \circ g(x) = \{(1, 1), (2, 1), (3, 1), (4, 1)\}$

ب) $f \circ f = \emptyset$

ج) $g \circ f(x) = \emptyset$

د) $g \circ g = \{(1, 1), (2, 1), (3, 1), (4, 1)\}$

$$f \circ g \Rightarrow f(g(x)) = \{(1, 1), (2, 1), (3, 1), (4, 1)\} \Rightarrow a = 1$$

$$g \circ f \Rightarrow g(f(x)) = \{(1, 1), (2, 1)\} \Rightarrow b = 1$$

$$\Rightarrow (a, b) \Rightarrow (1, 1)$$

$$f(x) = ax + b$$

$$f(ax+b) = a(ax+b) + b$$

(2) (E)

$$g(x) = cx + d$$

$$\Rightarrow \begin{cases} ax + ab + b \\ cx + r \end{cases}$$

$$\Rightarrow \begin{cases} ar = \varepsilon \Rightarrow a = \pm r \\ ab + b = r \Rightarrow \begin{cases} b = 1 \\ b = -r \end{cases} \end{cases}$$

$$\Rightarrow \begin{cases} f(x) = 4x + 1 \\ f(x) = -2x - 3 \end{cases}$$

$$g(f(x)) \Rightarrow g(f(-1))$$

$$f(-1) \Rightarrow \begin{cases} 2(-1) + 1 = -1 \\ -2(-1) - 3 = -1 \end{cases}$$

$$2x + 3 = -1 \Rightarrow 2x = -4$$

$$\Rightarrow g(-1) = 3 \times (-2) - 2 = -8$$

$$\Rightarrow x = -2$$

$$f(x) = \sqrt{x+|x|} \Rightarrow x+|x| \geq 0 \Rightarrow x \geq -|x| \Rightarrow |x| \geq x \quad (1) (V)$$

$$\Rightarrow D_f = \mathbb{R} \quad g(x) = \frac{1}{x^2 - \varepsilon x} \Rightarrow x^2 - \varepsilon x \neq 0 \Rightarrow D_g = \mathbb{R} - \{0, \varepsilon\}$$

$$D_{g \circ f} = \{x \in D_f \mid f(x) \in D_g\} \Rightarrow D_{g \circ f} = \{x \in \mathbb{R} \mid \sqrt{x+|x|} \neq \{0, \varepsilon\}\}$$

$$\Rightarrow D_{g \circ f} = \mathbb{R} - \{0, \varepsilon\}$$

$$D_f = [-1, 1] \Rightarrow 1 - x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1 \quad (2) (1)$$

$$D_g = [0, +\infty) \Rightarrow D_{f+g} = D_f \cap D_g = [0, 1]$$

$$D_{(f+g) \circ f} = \{x \in D_f \mid f(x) \in D_{f+g}\} = \{x \in [-1, 1] \mid 0 \leq \sqrt{1-x^2} \leq 1\}$$

$$0 \leq \sqrt{1-x^2} \leq 1 \Rightarrow 1-x^2 \leq 1 \Rightarrow x^2 \geq 0 \Rightarrow \text{جميع } x \text{ مقبول}$$

$$\Rightarrow D_{(f+g) \circ f} = [-1, 1]$$

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$$\text{iv) } f\left(\frac{3x+1}{x-3}\right) = 2x+5 \quad \frac{3x+1}{x-3} = t \Rightarrow x = \frac{3t+1}{t-3} \quad (9)$$

$$\Rightarrow f(t) = 2\left(\frac{3t+1}{t-3}\right) + 5 \Rightarrow f(t) = \frac{13t-11}{t-3} \quad (10)$$

$$\Rightarrow f(x) = \frac{13x-11}{x-3}$$

$$\therefore f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3} \Rightarrow \left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\frac{1}{x} + 3x$$

$$x + \frac{1}{x} = t$$

$$\Rightarrow x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(\frac{1}{x} + x\right)$$

$$\Rightarrow f(t) = t^3 - 3t \Rightarrow f(x) = x^3 - 3x$$

$$f(x) = x\sqrt{x} \Rightarrow \int x\sqrt{x} = 1 \Rightarrow x = 1 \quad (11) \quad (10)$$

$$\left\{ \begin{array}{l} \Rightarrow x-1=1 \\ x\sqrt{x} = 2 \Rightarrow x=2 \end{array} \right.$$

$$\begin{aligned} \sqrt{x+1} \neq 0 &\rightarrow x \neq -1 \\ \sqrt{x+1} \neq 1 &\rightarrow x \neq 0 \end{aligned} \rightarrow D_{\text{def}} = (0, +\infty) - \{1\}$$

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