

$$f(x) \begin{cases} \cot \frac{\pi x}{4} & ; x \leq 1 \quad (1) \\ \sqrt{x^2+1} & ; x > 1 \quad (2) \end{cases} \quad f\left(\frac{\sqrt{e}}{e}\right) = \cot \frac{\pi \times \frac{\sqrt{e}}{e}}{4} = \cot \frac{\pi}{6} = \sqrt{3}$$

$$f(\sqrt{e}) = \sqrt{(\sqrt{e})^2+1} = \sqrt{e+1} = \sqrt{4} = 2$$

یسی در رابطه (۱) جایگذاری می شود. $\frac{\sqrt{e}}{e} < 1$

یسی در رابطه (۲) جایگذاری می شود. $\sqrt{e} > 1$

$$\frac{\sqrt{e}}{e} \rightarrow \cot \frac{\pi}{6} \rightarrow \sqrt{3} \rightarrow (1)$$

$$a) \quad g\left(\frac{\pi}{e}\right) = 2 \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{4} \quad \left\{ \quad f\left(\frac{1}{4}\right) = \sqrt{\frac{4 \times \frac{1}{4} - 1}{e}} = \sqrt{\frac{e}{e}} = \left(\frac{\sqrt{e}}{e}\right) \right.$$

$$\frac{\pi}{e} \rightarrow \frac{1}{4} \rightarrow \frac{\sqrt{e}}{e}$$

$$b) \quad g(\sqrt{e}) = \frac{\sqrt{e}}{1-\sqrt{e}} = \frac{\sqrt{e}(1+\sqrt{e})}{-1} \Rightarrow -\sqrt{e}-2 \Rightarrow -(\sqrt{e}+2)$$

$$f(-(\sqrt{e}+2)) = [-\sqrt{e}-2] \Rightarrow (-4)$$

$$\sqrt{e} \rightarrow -\sqrt{e}-2 \rightarrow -4$$

$$f\left(\frac{\pi}{e}\right) = \sin \frac{\pi}{e} = \frac{\sqrt{e}}{e} \Rightarrow g(x) = x\sqrt{1-x^2} \Rightarrow g(x) = \frac{\sqrt{e}}{e} \sqrt{1-\left(\frac{\sqrt{e}}{e}\right)^2}$$

$$\frac{\sqrt{e}}{e} \sqrt{1-\frac{e}{e^2}} \Rightarrow \frac{\sqrt{e}}{e} \sqrt{\frac{e}{e^2}} \Rightarrow \frac{\sqrt{e}}{e} \times \frac{\sqrt{e}}{e} = \frac{e}{e^2} = \frac{1}{e}$$

$$\frac{\pi}{e} \rightarrow \frac{\sqrt{e}}{e} \rightarrow \frac{1}{e}$$

$$a) \quad f \circ g(x) \Rightarrow \left\{ \begin{array}{l} e \rightarrow d \rightarrow \emptyset \Rightarrow (e, \emptyset) \\ e \rightarrow e \rightarrow \emptyset \Rightarrow (e, \emptyset) \\ e \rightarrow a \rightarrow \emptyset \Rightarrow (e, \emptyset) \\ a \rightarrow 10 \rightarrow 12 \Rightarrow (a, 12) \\ 10 \rightarrow 12 \Rightarrow (10, 12) \end{array} \right\} \quad f \circ f(x) \left\{ \begin{array}{l} e \rightarrow \emptyset \rightarrow \emptyset \\ e \rightarrow d \rightarrow \emptyset \\ a \rightarrow 12 \rightarrow \emptyset \\ 10 \rightarrow 12 \rightarrow \emptyset \end{array} \right. \quad f \circ f(x) = \emptyset$$

$$b) \quad g \circ f(x) \Rightarrow \left\{ \begin{array}{l} e \rightarrow d \rightarrow x \\ e \rightarrow e \rightarrow x \\ a \rightarrow 12 \rightarrow x \\ 10 \rightarrow 12 \rightarrow x \end{array} \right. \quad g \circ f(x) = \emptyset$$

$$1) \quad g \circ g(x) \left\{ \begin{array}{l} e \rightarrow e \rightarrow 1 \Rightarrow (e, 1) \\ e \rightarrow e \rightarrow e \Rightarrow (e, e) \\ e \rightarrow a \rightarrow 10 \Rightarrow (e, 10) \\ a \rightarrow 10 \rightarrow \emptyset \end{array} \right. \quad g \circ g(x) = \emptyset$$

$$f \circ g(x) \quad (a, b) = (e, \emptyset)$$

$$\left. \begin{array}{l} 1 \rightarrow 2 \rightarrow 1 \quad (1, 1) \\ e \rightarrow 1 \rightarrow 7 \quad (e, 7) \\ a \rightarrow e \rightarrow 2 \quad (a, 2) \Rightarrow (e, 2) \\ b \rightarrow 1 \rightarrow 7 \quad (b, 7) \Rightarrow (1, 7) \end{array} \right\} \quad \boxed{a=e} \quad \boxed{b=\emptyset}$$

$$g \circ f(x) \left\{ \begin{array}{l} e \rightarrow 1 \rightarrow 2 \\ e \rightarrow 2 \rightarrow \emptyset \\ e \rightarrow \emptyset \rightarrow 1 \Rightarrow b=\emptyset \\ 1 \rightarrow 7 \rightarrow \emptyset \end{array} \right.$$

$$f(f(x)) = ex + e \Rightarrow f(ax+b) = a(ax+b) + b \Rightarrow a^2x + ab + b = ex + e$$

$$ex + e \Rightarrow \boxed{a=e} \quad \boxed{b=1}$$

$$g(x+e) = ex - e \Rightarrow a(x+e) + b = ex - e \Rightarrow ax + a + b = ex - e$$

$$ex - e \Rightarrow a=1, b=-e \quad g = 1 \cdot x - e$$

$$g(f(-1)) \Rightarrow g(-1) \Rightarrow \boxed{-1}$$

6

$$f(x) = \sqrt{x+|x|}$$

$$g(x) = \frac{1}{x^2 - ex} \Rightarrow x(x-e) \Rightarrow x \leq e$$

$$g \circ f = \{ x \in Df, f(x) \in Dg \}$$

$$\sqrt{x+|x|} = 0 \Rightarrow x+|x| \geq 0 \Rightarrow x \leq 0$$

$$x \leq 0, \sqrt{x+|x|} = e \Rightarrow \boxed{x=1}$$

$$Dg \circ f = \mathbb{R} - [(-\infty, 0] \cup \{1\}]$$

7

$$f(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x}$$

$$Dg = [0, +\infty) \Rightarrow Df \cap g = Df \cap g = [0, 1]$$

$$D(f+g) \cap f = \{ x \in Df, f(x) \in Dg \} \Rightarrow \{ x \in [-1, 1] \}$$

$$0 \leq \sqrt{1-x^2} \leq 1 \Rightarrow 1-x^2 \leq 1 \Rightarrow -x^2 \leq 0 \Rightarrow x^2 \geq 0$$

$$D(f+g) \cap f = [0, 1]$$

8

$$a) \quad \frac{ex+1}{x-e} = t \Rightarrow ex+1 = t(x-e) \Rightarrow x(et) = -et-1 \Rightarrow$$

$$x = \frac{et+1}{t-e} \Rightarrow f(t) = f\left(\frac{et+1}{t-e}\right) = \frac{1 \cdot x + e}{x - e} = \frac{1 \cdot \frac{et+1}{t-e} + e}{\frac{et+1}{t-e} - e}$$

$$b) \quad x + \frac{1}{x} = t \Rightarrow (t)^e = x^e + \frac{1}{x^e} + ex + \frac{e}{x} \Rightarrow \left(x + \frac{1}{x}\right)^e - \frac{e}{x} - ex =$$

$$f^e - et \Rightarrow \boxed{f(x) = x^e - ex}$$

9

$$g \circ f(x) \rightarrow g(f(x)) \rightarrow g(x\sqrt{x}) = 0$$

$$x\sqrt{x} = e\sqrt{e} \Rightarrow \boxed{x=e} \Rightarrow x\sqrt{x} = 1 \Rightarrow \boxed{x=1}$$

$$\Delta x = e-1 = \boxed{1}$$

10