ملیکا محمد حسینی

سوال یک ←

$$f(x)=\begin{cases} \cot\frac{\pi x}{4} & x\le 1 \\ \sqrt{x^2+1} & x>1 \end{cases} \qquad f\circ f\left(\frac{2}{3}\right)?$$

$$\cot\left(\frac{\frac{2}{3}\pi}{4}\right)\to \cot\left(\frac{\pi}{6}\right)=\sqrt{3}$$

$$f\left(\frac{2}{3}\right)=\sqrt{3} \qquad f\left(f\left(\frac{2}{3}\right)\right)=f(\sqrt{3})$$

$$\to \sqrt{(\sqrt{3})^2+1}=2 \longrightarrow f\circ f\left(\frac{2}{3}\right)=\boxed{2}$$

دو ←

$$f\left(\frac{1}{x}\right)=\sqrt{\frac{2x-1}{x^2}}$$

$$g(x)=2\cos^2(x)\longrightarrow g\left(\frac{\pi}{3}\right)=2\cos^2\left(\frac{\pi}{3}\right)\to 2\left(\frac{1}{2}\right)^2=\boxed{\frac{1}{2}}$$

$$f\circ g\left(\frac{\pi}{3}\right)=?\to f\left(g\left(\frac{\pi}{3}\right)\right)=?$$

$$f\left(\frac{1}{2}\right)=\sqrt{\frac{2\left(\frac{1}{2}\right)-1}{\left(\frac{1}{2}\right)^2}}=\sqrt{\frac{0}{\frac{1}{4}}}\to\sqrt{0}=0$$

$$f\circ g\left(\frac{\pi}{3}\right)=0$$

$$f(x)=[x] \quad , \quad g(x)=\frac{x}{1-x} \quad , \quad f\circ g(\sqrt{2})=?$$

$$f(g(\sqrt{2}))=?$$

$$g(\sqrt{2})=\frac{\sqrt{2}}{1-\sqrt{2}}\times\frac{1+\sqrt{2}}{1+\sqrt{2}}=\frac{\sqrt{2}\cdot(1+\sqrt{2})}{1-2}=\frac{\sqrt{2}+2}{-1}=-(\sqrt{2}+2)$$

$$\sqrt{2}\simeq 1.4 \qquad f(g(\sqrt{2}))=[-1.4-2]=[-3.4]=\boxed{-4}$$

سه ←

$$f(x)=\sin x \qquad g(x)=x\sqrt{1-x^2} \qquad g\circ f\left(\frac{\pi}{4}\right)=?$$

$$g\left(f\left(\frac{\pi}{4}\right)\right)\Rightarrow f\left(\frac{\pi}{4}\right)=\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$

$$g\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{2}\sqrt{\underbrace{1-\left(\frac{\sqrt{2}}{2}\right)^2}_{1-\frac{2}{4}\to\frac{2}{4}}}\longrightarrow\frac{\sqrt{2}}{2}\sqrt{\frac{2}{4}}=\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2}=\frac{2}{4}=\boxed{\frac{1}{2}}$$

چهار ←

$$f(x)=\{(4,5),(6,5),(8,12),(10,12)\}$$

$$g(x)=\{(4,6),(2,4),(6,8),(8,10)\}$$

$f\circ g\to$

$$4\xrightarrow{g(x)}6\xrightarrow{f(x)}5$$
$$2\to4\to5$$
$$6\to8\to12$$
$$8\to10\to12$$
$$\longrightarrow\{(4,5),(2,5),(6,12),(8,12)\}$$

$g\circ f\to$

$$4\to5\to\text{—}$$
$$6\to5\to\text{—}$$
$$8\to12\to\text{—}$$
$$10\to12\to\text{—}$$
$$\longrightarrow\varnothing$$

$f\circ f\to$

$$4\to5\to\text{—}$$
$$6\to5\to\text{—}$$
$$8\to12\to\text{—}$$
$$10\to12\to\text{—}$$
$$\longrightarrow\varnothing$$

$g\circ g\to$

$$4\to6\to8$$
$$2\to4\to6$$
$$6\to8\to10$$
$$8\to10\to\text{—}$$
$$\to\{(4,8),(2,6),(6,10)\}$$

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سوال ۵

$(f\circ g)(4)=2 \rightarrow f(\underbrace{g(4)}_{K})=2$

$f(K)=2 \rightarrow f(3)=2 \Longrightarrow g(4)=3 \Longrightarrow (4,3)\in g$

$(a,3),(b,1) \longrightarrow (a,3)$ همان $(4,3) \longrightarrow a=4$

$g\circ f(4)=1 \rightarrow g(\underbrace{f(4)}_{5})=1 \longrightarrow g(5)=1 \longrightarrow (b,1)$ همان $(5,1) \rightarrow b=5$

$(a,b)=(4,5)$

سوال ۶

$f(f(x))=4x+3$

$g(2x+3)=3x-2$

$g\circ f(-1)$

$f(x)=ax+b$

$g(x)=mx+n$

$f(ax+b)=a(ax+b)+b=a^2x+ab+b$

$a^2x+ab+b=4x+3$

$a^2=4 \rightarrow a=\pm 2$

$a=2 \rightarrow ab+b=3 \longrightarrow 2b+b=3 \rightarrow 3b=3 \rightarrow b=1 \longrightarrow 2x+1$

$a=-2 \longrightarrow -2b+b=3 \rightarrow -b=3 \rightarrow b=-3 \longrightarrow -2x-3$

$g(2x+3)=m(2x+3)+n \Rightarrow 2mx+3m+n=3x-2$

$2m=3 \rightarrow m=\frac{3}{2}$

$3m+n=-2 \rightarrow \frac{9}{2}+n=-2 \rightarrow n=-\frac{13}{2}$

$g(x)=\frac{3}{2}x-\frac{13}{2}$

$2x+1$

$\begin{cases} f(-1)=2(-1)+1=-1 \\ g(-1)=\frac{3}{2}(-1)-\frac{13}{2}=-8 \end{cases}$

$\boxed{g\circ f(-1)=-8}$

$-2x-3$

$\begin{cases} f(-1)=-1 \\ g(-1)=-8 \end{cases}$

سوال ۷

$f(x)=\sqrt{x+|x|}$ $\quad g(x)=\frac{1}{x^2-4x}$ $\quad D_{g\circ f}=?$

$g\circ f(x)=g(f(x))=\frac{1}{(f(x))^2-4f(x)}$

$x+|x|\geq 0$

$x\geq 0 \quad |x|=x \quad x+|x|=2x\geq 0$

$x<0 \quad |x|=-x \quad x+|x|=0$

$D_f = x\geq 0 \cup \{x<0 \mid x+|x|=0\} = \{0\}\cup\{0,+\infty) = [0,+\infty)$

$g\circ f(x)=\frac{1}{t^2-4t} \quad t\in[0,+\infty) \quad t^2-4t\neq 0 \quad t(t-4)\neq 0 \begin{cases} t\neq 0 \\ t\neq 4 \end{cases}$

$t=0 \rightarrow f(x)=0 \quad x=0$ حذف

$t=4 \rightarrow f(x)=4 \quad x=8 \quad \sqrt{2x}=4 \rightarrow 2x=16 \rightarrow x=8$ حذف ۸

$D_{g\circ f}=[0,+\infty)\setminus\{8\}$

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سوال ۸

$f(x)=\sqrt{1-x^2}$ $\quad g(x)=\sqrt{x}$ $\quad D_{(f+g)\circ f}=?$

$f(f(x))+g(f(x))=\sqrt{1-(f(x))^2}+\sqrt{f(x)}$

$1-(f(x))^2\geq 0$ $\quad f(x)=\sqrt{1-x^2}\longrightarrow (f(x))^2=1-x^2$

$1-(f(x))^2=1-(1-x^2)=x^2\geq 0$

$g(f(x))=\sqrt{f(x)}=\sqrt{\sqrt{1-x^2}}$

$\sqrt{1-x^2}\geq 0$ $\quad 1-x^2\geq 0\rightarrow -1\leq x\leq 1\rightarrow D=[-1,1]$

سوال ۹

$f(\underbrace{\frac{3x+1}{x-2}}_{t})=4x+5$

$tx-2t=3x+1$

$x(t-3)=2t+1$

$t=\frac{3x+1}{x-2}$

$x=\frac{2t+1}{t-3}\longrightarrow 4\left(\frac{2t+1}{t-3}\right)+5\rightarrow f(t)=\frac{8t+4}{t-3}+5$

$f(t)=\frac{8t+4}{t-3}+\frac{5(t-3)}{t-3}\longrightarrow \boxed{\frac{13x-11}{x-3}=f(x)}$

$f(x+\frac{1}{x})=x^3+\frac{1}{x^3}$

$t=x+\frac{1}{x}$

$x^3+\frac{1}{x^3}=(x+\frac{1}{x})^3-3(x+\frac{1}{x})\longrightarrow x^3+\frac{1}{x^3}=t^3-3t$

$f(t)=t^3-3t\longrightarrow f(x)=x^3-3x$

سوال ۱۰

$g(1)=0$ $\quad g(2\sqrt{2})=0$

$(g\circ f)(x)=g(f(x))=0\longrightarrow f(x)=g$

$f(x)=x^{\frac{3}{2}}=1$ $\quad x^{\frac{3}{2}}=2\sqrt{2}$

$x^{\frac{3}{2}}=1\longrightarrow x=1^{\frac{2}{3}}$

$x^{\frac{3}{2}}=2\sqrt{2}\longrightarrow x=(2\sqrt{2})^{\frac{2}{3}}$

$2\times 2^{\frac{1}{2}}=2^{\frac{3}{2}}\longrightarrow (2^{\frac{3}{2}})^{\frac{2}{3}}=2$

$g(f(x))=0$

$x=1$ $\quad x=2$

$2-1=\boxed{1}$