

$$f \circ f\left(\frac{1}{\sqrt{x}}\right) = f\left(f\left(\frac{1}{\sqrt{x}}\right)\right) = f\left(\cot \frac{\pi}{4}\right) = f(1) = \sqrt{1^2 + 1} = \sqrt{2} = 2$$

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الف) $f \circ g\left(\frac{\pi}{\sqrt{x}}\right) \rightarrow f\left(\frac{1}{x}\right) = \sqrt{\frac{2x-1}{x^2}} \Rightarrow f(x) = \sqrt{\frac{\frac{x}{x} - 1}{\left(\frac{1}{x}\right)^2}} = \sqrt{\frac{x^2(2-x)}{x}}$

$$= \sqrt{2x-x^2} \Rightarrow f\left(g\left(\frac{\pi}{\sqrt{x}}\right)\right) = f\left(2 \cos^2 \frac{\pi}{4}\right) = f\left(2 \times \frac{1}{2}\right) = f\left(\frac{1}{2}\right) =$$

$$\sqrt{2\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^2} = \sqrt{\frac{4}{4} - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

ب) $f \circ g(\sqrt{2}) = f(g(\sqrt{2})) = f\left[\frac{\sqrt{2}}{1-\sqrt{2}}\right] = f\left[\frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}\right] = f\left[\frac{\sqrt{2}(1+\sqrt{2})}{-1}\right] =$
 $[-2, 2] = -2$

$$f(x) = \sin x, \quad g(x) = x \sqrt{1-x^2}$$

$$f\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = \frac{\sqrt{2}}{2} \Rightarrow g \circ f\left(\frac{\pi}{2}\right) = g\left(f\left(\frac{\pi}{2}\right)\right) = g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} =$$

$$\frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$f(x) = \{(4, 5), (4, 5), (1, 12), (1, 12)\}, \quad g(x) = \{(4, 4), (2, 4), (4, 1), (1, 1)\}$$

الف) $f \circ g(x) = \{(4, 5), (2, 5), (4, 12), (1, 12)\}$

ب) $g \circ f(x) = \emptyset$

ج) $f \circ f(x) = \emptyset \quad \text{د) } g \circ g(x) = \{(4, 1), (4, 1)\}$

$$f = \{(2, 1), (3, 2), (4, 5), (1, 6)\}, \quad g = \{(1, 2), (3, 1), (a, 5), (b, 1)\}$$

$$(4, 2) \in f \circ g \Rightarrow f(g(4)) = 2 \rightarrow a = 4$$

$$(4, 1) \in g \circ f \rightarrow g(f(4)) = 1 \rightarrow b = 5 \rightarrow (a, b) = (4, 5)$$

$$f(f(x)) = 2x + 3, \quad g(2x + 3) = 3x - 2$$

$$f(x) = ax + b \rightarrow f(f(x)) = f(ax + b) = a(ax + b) + b$$

$$\left. \begin{aligned} f \circ f(x) &= a^2x + ab + b \\ f \circ f(x) &= 2x + 3 \end{aligned} \right\} \rightarrow \begin{cases} a^2 = 2 \rightarrow a = \sqrt{2} \\ ab + b = 3 \rightarrow \sqrt{2}b + b = 3 \Rightarrow b = 1 \end{cases} \Rightarrow f(x) = \sqrt{2}x + 1$$

$$g \circ f(-1) = g(f(-1)) = 3(-2) - 2 = -8$$

$$2x + 3 = -1 \rightarrow 2x = -4 \Rightarrow x = -2$$

$$f(x) = \sqrt{x+1} \quad , \quad g(x) = \frac{1}{x^2-4x}$$

$$D_{g \circ f} = \{x \in D_f \mid f(x) \in D_g\} \stackrel{*}{=}$$

$$D_f: x+1 \geq 0 \Rightarrow x \geq -1 \rightarrow D_f = \mathbb{R}$$

$$D_g: x^2-4x \neq 0 \Rightarrow x(x-4) \neq 0 \Rightarrow x \neq 0, 4$$

$$\stackrel{*}{=} \{x \in \mathbb{R} \mid \sqrt{x+1} \neq 0, 4\} = (0, +\infty) - \{4\}$$

$$\begin{cases} \sqrt{x+1} \neq 0 \xrightarrow{\text{فان}} x+1 \neq 0 \Rightarrow x \neq -1 \rightarrow x \in (0, +\infty) \\ \sqrt{x+1} \neq 4 \Rightarrow x+1 \neq 16 \Rightarrow x \neq 15 \end{cases}$$

$$f(x) = \sqrt{1-x^2}, \quad g(x) = \sqrt{x}$$

$$D_f: 1-x^2 \geq 0 \rightarrow x^2 \leq 1 \rightarrow |x| \leq 1 \rightarrow -1 \leq x \leq 1 \rightarrow D_f = [-1, 1]$$

$$D_g: x \geq 0 \Rightarrow D_f \cap D_g = [0, 1]$$

$$D_{(f+g) \circ f} = \{x \in D_f \mid f(x) \in D_f \cap D_g\} = \{x \in [-1, 1] \mid \sqrt{1-x^2} \in [0, 1]\} \stackrel{*}{=}$$

$$\sqrt{1-x^2} \in [0, 1] \rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \xrightarrow{\text{فان}} 0 \leq 1-x^2 \leq 1 \Rightarrow -1 \leq -x^2 \leq 0 \xrightarrow{x^2 \geq 0} 0 \leq x^2 \leq 1$$

$$\Rightarrow \sqrt{x^2} \leq 1 \rightarrow |x| \leq 1 \rightarrow -1 \leq x \leq 1$$

$$\stackrel{*}{=} \{x \in [-1, 1] \mid x \in [-1, 1]\} = [-1, 1]$$

$$\text{الف) } f\left(\frac{x+1}{x-2}\right) = x+5 \quad \frac{x+1}{x-2} = t \Rightarrow x+1 = t(x-2) \Rightarrow$$

$$x - tx = -2t - 1 \Rightarrow x(1-t) = -2t - 1 \Rightarrow x = \frac{-2t-1}{1-t} \Rightarrow$$

$$f(t) = f\left(\frac{-2t-1}{1-t}\right) + 5 \Rightarrow f(x) = f\left(\frac{x+1}{x-2}\right) + 5$$

$$\text{ب) } f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} \quad x + \frac{1}{x} = t$$

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} \Rightarrow \left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$$

$$\left(x + \frac{1}{x}\right)^2 - 2 = x^2 + \frac{1}{x^2} \Rightarrow f(t) = t^2 - 2 \Rightarrow f(x) = x^2 - 2$$

$$g(1) = 0, \quad g(\sqrt{x}) = 0, \quad f(x) = x\sqrt{x}$$

$$g \circ f(x) = 0 \rightarrow g(f(x)) = 0 \Rightarrow f(x) = 1 \Rightarrow x\sqrt{x} = 1 \xrightarrow{\text{فان}} x^3 = 1 \Rightarrow x = 1$$

$$f(x) = x\sqrt{x} \rightarrow x\sqrt{x} = x\sqrt{x} \Rightarrow x = x \quad x-1=1$$