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آرک تانگ یا دایره
یازدهم فصل ۱
تکلیف مشابه ۹

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases}$$

$$\cot \frac{\pi}{4} = \sqrt{3} \rightarrow \sqrt{(\sqrt{3})^2 + 1} = \sqrt{4} = 2$$

$\Rightarrow f \circ f\left(\frac{1}{2}\right) = 2$

۱) $r \cos^2\left(\frac{\pi}{r}\right) = r\left(\frac{1}{r}\right)^2 = \frac{1}{r} \rightarrow \sqrt{\frac{r(r)-1}{(r)^2}} = \sqrt{\frac{r}{r}} = \frac{\sqrt{r}}{r}$

$f \circ g\left(\frac{\pi}{r}\right) = \frac{\sqrt{r}}{r}$

۲) $\frac{\sqrt{r}}{1-\sqrt{r}} = \frac{\sqrt{r}+r}{1-r} = -\sqrt{r}-r \rightarrow [-\sqrt{r}-r] = -4 \quad f \circ g(\sqrt{r}) = -4$

$\sin \frac{\pi}{r} = \frac{\sqrt{r}}{r} \rightarrow \frac{\sqrt{r}}{r} \sqrt{1 - \left(\frac{\sqrt{r}}{r}\right)^2} = \frac{1}{r} \quad g \circ f\left(\frac{\pi}{r}\right) = \frac{1}{r}$

۳) $f \circ g(m) = \{(r, \omega), (4, \omega), (4, 12), (2, 12)\}$

۴) $g \circ f(m) = \{ \}$

۵) $f \circ f(m) = \{ \}$

۶) $g \circ g(m) = \{(2, 4), (4, 1), (4, 1), (2, 4)\}$

$(4, 2) \in f \circ g \quad (a, r) \rightarrow (r, 2) \rightarrow a = 4 \quad (a, b) = (4, \omega)$

$(4, 1) \in g \circ f \quad (4, \omega) \rightarrow (b, 1) \rightarrow b = \omega$

$f(f(m)) = a(an+b)+b = a^2n+ab+b = 4n+3 \quad a^2 = 4 \rightarrow a = \pm 2$

$a = +2 \rightarrow 2b = 3 \rightarrow b = 1.5 \rightarrow f(m) = 2m+1.5 \quad m = -1 \rightarrow 2(-1)+1.5 = -0.5$

$a = -2 \rightarrow -2b = 3 \rightarrow b = -1.5 \rightarrow f(m) = -2m-1.5 \quad m = -1 \rightarrow -2(-1)-1.5 = 0.5 \Rightarrow f(-1) = -1$

$2m+3 = -1 \rightarrow 2m = -4 \rightarrow m = -2 \rightarrow g(-1) = 3(-2)-2 = -8 \quad g \circ f(-1) = -8$

$D_{g \circ f} = \{n \in D_f ; f(m) \in D_g\} \quad D_f = x+|m| \geq 0 = \mathbb{R} \quad D_g = x^2 \cdot \frac{x-1}{x} \neq 0 \rightarrow x \neq 0, x \neq 1$

$\{x \in \mathbb{R} : -\sqrt{x+|m|} \in \mathbb{R} - \{0, 1\}\} \rightarrow D_{g \circ f} = (0, +\infty) - \{1\}$

s.a.m

$$D_{(f+g) \circ f} = \{x \in D_f : f(x) \in D_{f+g}\} \quad D_f = 1 - x^r \geq 0 \rightarrow 1 \geq x^r \rightarrow -1 \leq x \leq 1 \quad \text{A}$$

$$D_{f+g} : \sqrt{x} + \sqrt{1-x^r} \rightarrow x \geq 0 \quad \text{A} \quad -1 \leq x \leq 1 \rightarrow D_{f+g} = [0, 1] \quad \text{r}$$

$$D_{(f+g) \circ f} = \{x \in [-1, 1] : x \in [-1, 1]\} \Rightarrow D_{(f+g) \circ f} = [-1, 1]$$

$$\text{ii) } f\left(\frac{r_{n+1}}{n-r}\right) = f_{n+1} + \omega \rightarrow \frac{r_{n+1}}{n-r} = t \rightarrow t(n-r) = r_{n+1} \rightarrow t_{n-r}t = r_{n+1} \quad \text{A}$$

$$\rightarrow r_{n-r}t_{n-r} = r_{n+1} \rightarrow n(t-r) = rt+1 \rightarrow n = \frac{rt+1}{t-r} \quad \text{r}$$

$$\rightarrow f_{n+1} + \omega = f\left(\frac{rt+1}{t-r}\right) + \omega \rightarrow f_{n+1} + \omega = \frac{1+t+r+\omega(t-r)}{t-r} = \frac{1+t-11}{t-r} \rightarrow f(t) = \frac{1+t-11}{t-r}$$

$$f(n) = \frac{1+r_{n+1}-11}{n-r} = \frac{1+r+r}{n-r} + \omega$$

$$\text{iii) } f\left(n + \frac{1}{n}\right) = n^r + \frac{1}{n^r} \quad n^r + \frac{1}{n^r} = \left(n + \frac{1}{n}\right)^r - r\left(n + \frac{1}{n}\right)$$

$$\Rightarrow f(n) = n^r - rn$$

$$g(1) = 0 \quad g(r\sqrt{r}) = 0 \quad g \circ f(n) = g(f(n)) \Rightarrow f(n) = 1, f(n) = r\sqrt{r} \quad \text{A}$$

$$\rightarrow n\sqrt{n} = 1 \rightarrow n = 1 \quad n\sqrt{n} = r\sqrt{r} \rightarrow n = r \quad g \circ f = \{(1, 0), (r, 0)\}$$

$$r-1=1$$