

نام و نام خانوادگی: فاطمه سنجی

پایه هفتم شماره: ۹

کلاس: یازدهم ریاضی
A

$$f\left(\frac{\sqrt{3}}{2}\right) = \cot \frac{\frac{\sqrt{3}}{2}\pi}{\frac{\sqrt{3}}{2}} = \cot \frac{\pi}{2} = \sqrt{3}$$

$$f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = \sqrt{4} = \pm 2 = f \circ f\left(\frac{\sqrt{3}}{2}\right)$$

الف) $f(x) = \sqrt{\frac{1-x}{1+x}}$ بجای x ضابطه $\frac{1}{x}$ می‌زنیم $\Rightarrow \sqrt{\frac{1-\frac{1}{x}}{1+\frac{1}{x}}} = \sqrt{\frac{x-1}{x+1}} = f(x)$

$$g(x) = 1 + \frac{\cos 2x}{2} = \frac{1 + \cos 2x}{2} \rightarrow g\left(\frac{\pi}{2}\right) = \frac{1}{2}, f\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$$

ب) $f \circ g(x) = \left[\frac{x}{1-x}\right] \Rightarrow f \circ g(\sqrt{2}) = \left[\frac{\sqrt{2}}{1-\sqrt{2}}\right] = [-\sqrt{2}-2] = [-4]$

$$g \circ f(x) = \sin x \sqrt{1 - \sin^2 x} = \sin x |\cos x|$$

$$g \circ f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} \times \left|\cos \frac{\pi}{2}\right| = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{1}{2}$$

الف) $f \circ g(x) = \{(2, 0), (2, 0), (4, 1), (1, 1)\}$

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$(2, 2) \in f \circ g \rightarrow \begin{matrix} \uparrow \\ \text{a} \end{matrix} \xrightarrow{g(x)} 2 \xrightarrow{f(x)} 2 \Rightarrow \boxed{a=2}$

$(2, 1) \in g \circ f \rightarrow 2 \xrightarrow{f(x)} \begin{matrix} \downarrow \\ \text{b} \end{matrix} \xrightarrow{g(x)} 1 \Rightarrow \boxed{b=0}$

$(a, b) \text{ مرتب شده } = (2, 0)$

$$f(x) = ax + b \Rightarrow f(f(x)) = a(ax + b) + b = a^2x + ab + b = \varepsilon x + \mu \rightarrow \boxed{a=2} \boxed{b=1}$$

$$\boxed{f(x) = 2x + 1}$$

$$2x + \mu = t \Rightarrow \frac{t - \mu}{2} = x \Rightarrow f(t) = 2\left(\frac{t - \mu}{2}\right) + 1 \Rightarrow \frac{2t - 9 - \varepsilon}{2} = \boxed{\frac{2t - 13}{2} = g(t)}$$

$$g\left(\frac{f(-1)}{-1}\right) = \frac{-3 - 13}{2} = \boxed{-8}$$

$$f(x) = \begin{cases} \sqrt{x} & x \geq 0 \\ 0 & x < 0 \end{cases} \quad g(x) = \frac{1}{x^2 - \varepsilon x} \quad *$$

$$\hookrightarrow D_g = \mathbb{R} - \{0, \varepsilon\} \rightarrow f(x) \neq 0 \text{ و } \varepsilon \text{ از مخرج صفر می شود}$$

$$\hookrightarrow f(x) = \varepsilon \Rightarrow \sqrt{2x} = \varepsilon \rightarrow \boxed{x = 1}$$

$$g \circ f \text{ معنی ندارد} \quad D_{g \circ f} = (0, 1) \cup (1, +\infty)$$

$$f(x) = \sqrt{1-x^2} \Rightarrow D_f = [-1, 1] \rightsquigarrow R_f = [0, 1] \rightsquigarrow \text{مجموعه های تابع}$$

$$g(x) = \sqrt{x} \Rightarrow D_g = [0, 1]$$

$$(f+g) \text{ از بازه } [0, 1] \text{ هستند}$$

$$\Rightarrow (f+g) \circ f = \sqrt{1+x^2} + \sqrt{1-\sqrt{1-x^2}} \rightarrow D_{(f+g) \circ f} = \boxed{[-1, 1]}$$

$$\frac{2x+1}{x-2} = t \Rightarrow tx - 2x = 1 + 2t \Rightarrow x = \frac{1+2t}{t-2}, f(t) = \varepsilon\left(\frac{1+2t}{t-2}\right) + \mu = \frac{13t-11}{t-2} \Rightarrow f(x) = \frac{13x-11}{x-2}$$

$$x + \frac{1}{x} = t \Rightarrow t^2 = \frac{1}{x^2} + x^2 + 2 + \frac{1}{x^2} \Rightarrow f(t) = t^2 - 2\left(x + \frac{1}{x}\right) = t^2 - 2t \Rightarrow f(x) = x^2 - 2x$$

$$g(x) = (x - 2\sqrt{x})(x-1) \quad \left\{ \begin{array}{l} g \circ f(x) = (x\sqrt{x} - 2\sqrt{x})(x\sqrt{x} - 1) \rightarrow 2, 1 = \text{مخرج} \\ f(x) = x\sqrt{x} \end{array} \right.$$

$$x\sqrt{x} = 2\sqrt{x} \quad x\sqrt{x} = 1 \rightarrow \boxed{x=1}$$

$$2-1 = \boxed{1} = \text{مخرج}$$