

الف) $6x^2 - 5x - 4 = 0 \rightarrow x^2 - 5x - 34 = 0 \rightarrow (x+4)(x-9) = 0 \rightarrow \begin{cases} x = -\frac{4}{1} = -4 \\ x = \frac{9}{1} = 9 \end{cases}$

ب) $3x^2 - 11x - 4 = 0 \rightarrow x^2 - 11x - 4 = 0 \rightarrow (x+1)(x-9) = 0 \rightarrow \begin{cases} x = -\frac{1}{1} = -1 \\ x = \frac{9}{1} = 9 \end{cases}$

ج) $4x^2 - 4x - 12 = 0 \rightarrow x^2 - 4x - 12 = 0 \rightarrow (x+2)(x-6) = 0 \rightarrow \begin{cases} x = -\frac{2}{1} = -2 \\ x = \frac{6}{1} = 6 \end{cases}$

د) $2x^2 - 5x - 12 = 0 \rightarrow x^2 - 5x - 12 = 0 \rightarrow (x+3)(x-8) = 0 \rightarrow \begin{cases} x = -\frac{3}{1} = -3 \\ x = \frac{8}{1} = 8 \end{cases}$

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الف) $x^2 - 5x - 14 = 0 \rightarrow (x+2)(x-7) = 0 \rightarrow \begin{cases} x = -2 \\ x = 7 \end{cases}$

ب) $2x^2 + 5x + 3 = 0 \rightarrow x^2 + 5x + 4 = 0 \rightarrow (x+1)(x+4) = 0 \rightarrow \begin{cases} x = -\frac{1}{1} = -1 \\ x = -\frac{4}{1} = -4 \end{cases}$

ج) $x^2 + 2x + 3 = 0 \rightarrow x = \frac{-2 \pm \sqrt{4-12}}{2} \Rightarrow \Delta < 0$ ریشه ندارد

د) $3x^2 + 13x - 2 = 0 \rightarrow x = \frac{-13 \pm \sqrt{169+24}}{6} \Rightarrow x = \frac{-13 \pm \sqrt{193}}{6}$

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$A = \{1, 2, 3\} / B = \{2, 3\}$

$A \times B = \{(1,2), (1,3), (2,2), (2,3), (3,2), (3,3)\}$

$B^2 = \{(2,2), (2,3), (3,2), (3,3)\}$

$\ominus \Rightarrow \{(1,2) \text{ و } (1,3)\}$

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الف) $f(x) = \{(3, m^2), (2, 1), (-3, m), (-2, m), (3, m+2), (m, 4)\}$

$\star: m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \Rightarrow (m+1)(m-2) = 0 \rightarrow \begin{cases} m = -1 \\ m = 2 \end{cases}$

$m = -1 \rightarrow f(x) = \{(2, 1), (2, 1), (-3, -1), (-2, -1), (3, 1), (-1, 4)\}$ غلط

$m = 2 \rightarrow f(x) = \{(3, 4), (2, 1), (-3, 2), (-2, 2), (3, 4), (2, 4)\}$ غلط

ب) $f(x) = \{(3, m^2), (2, 1), (m, 4), (3, m+2), (2, m), (-1, 3)\}$

$\star: m^2 = m+2 \rightarrow (m+1)(m-2) = 0 \rightarrow \begin{cases} m = -1 \\ m = 2 \end{cases}$

$m = -1 \rightarrow f(x) = \{(3, 1), (2, 1), (-1, 4), (2, 1), (-2, 1), (-1, 3)\}$ غلط

$m = 2 \rightarrow f(x) = \{(3, 4), (2, 1), (2, 4), (3, 4), (2, 4), (2, 4)\}$ غلط

ج) $f(x) = \{(1, 2), (1, 3), (2, 4)\}$ غلط

د) $f(x) = \{(1, 4), (1, 4), (4, 1)\}$ غلط

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$f(x) = \{(1, 2), (1, 3), (2, 4)\}$

$a^2 = 3a - 2 \Rightarrow a^2 - 3a + 2 = 0 \rightarrow (a-1)(a-2) = 0 \rightarrow \begin{cases} a = 1 \\ a = 2 \end{cases}$

$a = 1 \rightarrow f(x) = \{(1, 1), (1, 1), (1, 1)\}$ غلط

$a = 2 \rightarrow f(x) = \{(1, 4), (1, 4), (4, 1)\}$ غلط

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$$f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ ax + b & 1 \leq x \leq 2 \\ x^2 & x \geq 2 \end{cases}$$

$$\textcircled{1}: x=1 \Rightarrow 0 = a+b \rightarrow -a=b$$

$$\textcircled{2}: x=2 \Rightarrow 0 = 4a + b = 1 \xrightarrow{\textcircled{1}} 4a - a = 1$$

$$a = 1$$

$$b = -1$$

$$a+b = -1 \times 1 = -1$$

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$$f(x) = \begin{cases} 2x - 3 & x \geq k \\ 5 - 3x & x \leq k \end{cases}$$

$$x=k \Rightarrow 2k - 3 = 5 - 3k$$

$$5k = 8$$

$$k = \frac{8}{5} \Rightarrow k = x \rightarrow k = \frac{8}{5}$$

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$$\text{الف) } y'' + y = x^2 - 4x + 3 \rightarrow y'' - y = 0$$

$$\Rightarrow |x| + |y| = 3 \xrightarrow{x=1} |y| = 2 \rightarrow y = \pm 2$$

$$\text{ج) } |y| = \sqrt{1 - \cos x} \xrightarrow{x=\frac{\pi}{2}} |y| = \sqrt{1 - 0} = 1 \rightarrow y = \pm 1$$

$$\Rightarrow y'' + x^2 + 2x + 1 = 0 \rightarrow y'' + (x+1)^2 = 0 \rightarrow y=0$$

$$x+1=0 \rightarrow x=-1$$

تابع است

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$$\text{الف) } \sqrt[3]{y} + x^2 = 2x + 2 \rightarrow \sqrt[3]{y} = -x^2 + 2x + 2 \rightarrow \begin{cases} \sqrt[3]{y_1} = x^2 + 2x + 2 \\ \sqrt[3]{y_2} = -x^2 + 2x + 2 \end{cases}$$

$$\Rightarrow \sqrt[3]{y_1} = \sqrt[3]{y_2} \rightarrow y_1 = y_2$$

$$\Rightarrow y'' + 2y = \omega x \rightarrow \begin{cases} y_1'' + 2y_1 = \omega x \\ y_2'' + 2y_2 = \omega x \end{cases} \Rightarrow y_1'' + 2y_1 = y_2'' + 2y_2 \rightarrow y_1'' - y_2'' = 2(y_2 - y_1)$$

$$(y_1 - y_2)(y_1' + y_2') = 2(y_2 - y_1) \Rightarrow (y_1 - y_2)(y_1' + y_2') + 2(y_1 - y_2) = 0 \Rightarrow (y_1 - y_2)(y_1' + y_2' + 2) = 0$$

$$\begin{cases} y_1 = y_2 \\ y_1' + y_2' + 2 = 0 \end{cases} \Rightarrow \Delta = y_1' - 4y_2' - 1 \rightarrow -4y_2' - 1 \rightarrow \Delta \neq 0 \rightarrow \text{تابع است}$$

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$$\text{الف) } x \sin y = y \sin x \xrightarrow{x=0} 0 = 0 \rightarrow \text{در مقدار ۰ صدق می کند}$$

پس تابع نیست

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$$\Rightarrow \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2 \xrightarrow{\sqrt{\frac{x}{y}} = \frac{1}{a}} a + \frac{1}{a} = 2 \xrightarrow{x^4} a^2 - 2a + 1 = 0 \rightarrow (a-1)^2 = 0 \rightarrow a=1$$

$$\rightarrow \frac{x}{y} = 1 \rightarrow x=y$$

تابع است