

الف) $4x^2 - 5x - 9 = 0 \rightarrow x^2 - 5x - 24 = 0 \rightarrow (x+4)(x-9) = 0 \rightarrow x_1 = -4, x_2 = 9$
 ب) $3x^2 - 11x - 2 = 0 \rightarrow x^2 - 11x - 2 = 0 \rightarrow (x+1)(x-9) = 0 \rightarrow x_1 = -1, x_2 = 9$
 ج) $4x^2 - 4x - 3 = 0 \rightarrow x^2 - 4x - 12 = 0 \rightarrow (x+2)(x-6) = 0 \rightarrow x_1 = -2, x_2 = 6$
 د) $2x^2 - 5x - 12 = 0 \rightarrow x^2 - 5x - 24 = 0 \rightarrow (x+4)(x-9) = 0 \rightarrow x_1 = -4, x_2 = 9$

الف) $x^2 - 5x - 14 = 0 \rightarrow (x+2)(x-7) = 0 \rightarrow x_1 = -2, x_2 = 7$
 ب) $2x^2 + 5x + 2 = 0 \rightarrow 2x^2 + 2x + 3 = 0 \Rightarrow x_1 = -1, x_2 = -\frac{3}{2}$
 ج) $x^2 + 2x + 3 = 0 \rightarrow \Delta = 4 - 12 = -8 \Rightarrow$ ریشه ندارد
 د) $3x^2 + 4x - 2 = 0 \rightarrow \Delta = 16 - 4(-2)(3) = 40 \Rightarrow x = \frac{-4 \pm \sqrt{40}}{6}$

$A \times B = \{(1,2)(1,3)(2,2)(2,3)(3,2)(3,3)\}$ $B^2 = \{(2,2)(2,3)(3,2)(3,3)\}$
 $A \times B - B^2 = \{(1,2)(1,3)\}$

الف) $f(x) = \{(2,m^2), (2,1), (-2,m), (-2,m), (2,m+2), (m,4)\}$
 $m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \rightarrow (m+1)(m-2) = 0 \rightarrow m = -1 \rightarrow \{(2,1), (2,1), (-2,-1), (-2,-1), (2,1), (-1,4)\}$
 ب) $g(x) = \{(2,m^2), (2,1), (m,4), (2,m+2), (-2,m), (-1,3)\}$
 $m^2 = m+2 \rightarrow m^2 - m - 2 = 0 \rightarrow (m+1)(m-2) = 0 \rightarrow m = -1 \rightarrow \{(-1,4), (-1,3)\}$
 $m = 2 \rightarrow \{(2,4), (2,1), (2,4), (2,4), (-2,2), (-1,3)\}$

$f(x) = \{(1,a^2), (1,3a-2), (a^2, 3a+2)\}$ $a^2 = 3a-2 \rightarrow a^2 - 3a + 2 = 0 \rightarrow a_1 = 1, a_2 = 2$
 $a = 1 \rightarrow f(x) = \{(1,1), (1,1), (1,5)\}$ $a = 2 \rightarrow f(x) = \{(1,4), (1,4), (4,8)\}$

$f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ ax + b & 1 < x \leq 2 \\ x^2 & x > 2 \end{cases}$ $x = 1 \rightarrow 0 = a + b, x = 2 \rightarrow 1a + b = 1$
 $\begin{cases} a + b = 1 \\ a + b = 0 \end{cases} \Rightarrow \begin{cases} a = 1 \\ b = -1 \end{cases} \Rightarrow a \times b = 1 \times -1 = -1$

$f(x) = \begin{cases} 2x - 3 & x \geq k \\ 4 - 3x & x \leq k \end{cases}$ $x = k \rightarrow 2k - 3 = 4 - 3k \Rightarrow 2k + 3k = 4 + 3 \Rightarrow 5k = 7 \Rightarrow k = \frac{7}{5}$

$$\text{الف) } y^3 - 4y = x^3 - 4x + 3 \rightarrow \begin{cases} y_1^3 - 4y_1 = x_1^3 - 4x_1 + 3 \\ y_2^3 - 4y_2 = x_2^3 - 4x_2 + 3 \end{cases} \xrightarrow{x_1 = x_2} y_1^3 - 4y_1 = y_2^3 - 4y_2 \Rightarrow$$

$$y_1^3 - y_2^3 - 4(y_1 - y_2) = 0 \rightarrow (y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2 - 4) = 0 \rightarrow \textcircled{1} y_1 - y_2 = 0 \rightarrow y_1 = y_2$$

$$\textcircled{2} y_1^2 + y_1 y_2 + y_2^2 - 4 = 0 \rightarrow y_1 = \frac{-y_2 \pm \sqrt{\Delta}}{2} = \frac{-y_2 \pm \sqrt{y_2^2 - 4}}{2} = \frac{-y_2 \pm \sqrt{y_2^2 - 4}}{2} \rightarrow \Delta = y_2^2 - 4 \rightarrow \Delta = y_2^2 - 4 \rightarrow \Delta = y_2^2 - 4 \rightarrow \Delta = y_2^2 - 4$$

چون کاهی دلتا مثبت است پس ستادی دوم کاهی برقرار است $\Rightarrow$ تابع نیست

$$\text{ب) } |x| + |y| = 3 \rightarrow x = 0 \rightarrow |y| = 3 \rightarrow y = \pm 3 \quad \text{تابع نیست}$$

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$$\text{ج) } |y| = \sqrt{1 - \cos x} \rightarrow 1 - \cos x \geq 0 \rightarrow 1 \geq \cos x \Rightarrow -1 \leq \cos x \leq 1 \Rightarrow \cos x = 1 \Rightarrow$$

$$|y| = 0 \Rightarrow (0, 0) \text{ تابع هست}$$

$$\text{د) } y^2 + x^2 + 2x + 1 = 0 \rightarrow y^2 + (x+1)^2 = 0 \quad (-1, 0) \quad \text{تابع نیست}$$

$$\text{الف) } \sqrt[3]{y} + x^2 = 2x - 2 \rightarrow \begin{cases} \sqrt[3]{y_1} + x_1^2 = 2x_1 - 2 \\ \sqrt[3]{y_2} + x_2^2 = 2x_2 - 2 \end{cases} \xrightarrow{x_1 = x_2} \sqrt[3]{y_1} = \sqrt[3]{y_2} \Rightarrow y_1 = y_2 \quad \text{تابع هست}$$

$$\text{ب) } y^3 + 2y = 5x \rightarrow \begin{cases} y_1^3 + 2y_1 = 5x_1 \\ y_2^3 + 2y_2 = 5x_2 \end{cases} \xrightarrow{x_1 = x_2} y_1^3 + 2y_1 = y_2^3 + 2y_2 \rightarrow y_1^3 - y_2^3 = 2y_2 - 2y_1 \Rightarrow$$

$$(y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2) + 2(y_2 - y_1) = 0 \rightarrow (y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2 + 2) = 0 \Rightarrow$$

$$\textcircled{1} y_1 - y_2 = 0 \rightarrow y_1 = y_2 \quad \textcircled{2} y_1 = \frac{-y_2 \pm \sqrt{\Delta}}{2} \rightarrow \Delta = y_2^2 - 4(y_2^2 + 2) \rightarrow \Delta = -3y_2^2 - 4 \rightarrow \Delta < 0 \rightarrow \text{تابع نیست}$$

$$\text{الف) } x \sin y = y \sin x \rightarrow x = 0 \rightarrow \sin y = 0 \Rightarrow y = 0^\circ, y = 180^\circ \quad \text{تابع نیست}$$

$$\text{ب) } \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2 \quad a = \sqrt{\frac{x}{y}}, b = \sqrt{\frac{y}{x}} \quad ab = \sqrt{\frac{x}{y} \times \frac{y}{x}} = 1$$

$$a + b = 2 \Rightarrow a = b = 1 \quad \sqrt{\frac{x}{y}} = 1 \Rightarrow \frac{x}{y} = 1 \Rightarrow x = y \quad \text{تابع هست}$$

شرط $(x, y) \neq (0, 0)$ یادداشت نه!

ب-۴) $(3, m^2)$ و $(3, m+2) \leadsto m^2 = m+2 \leadsto m^2 - m - 2 = 0$

$m=2 \rightarrow (2, 1)$ و $(2, 4) \times$

$m=-1 \rightarrow (-1, 4) (-1, 1) \times$ به ازای هیچ مقدار m عبارت تانژانت!

ج-۸ $|y| = \sqrt{1 - \cos x} \xrightarrow{x = \frac{\pi}{2}} |y| = \sqrt{1 - 0} \leadsto |y| = 1$

$\leadsto y = \pm 1$

به ازای $x = \frac{\pi}{2}$ ، در y صفت داریم پس تانژانت

(باز هم مثال نقض زدیم!)

ب-۱۰) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2$

این عبارت تنها زمانی به قدر است که $\sqrt{\frac{x}{y}} = 1$ باشد پس

$\sqrt{\frac{x}{y}} = 1 \leadsto \frac{x}{y} = 1 \leadsto x = y$ و $(x, y \neq 0)$

