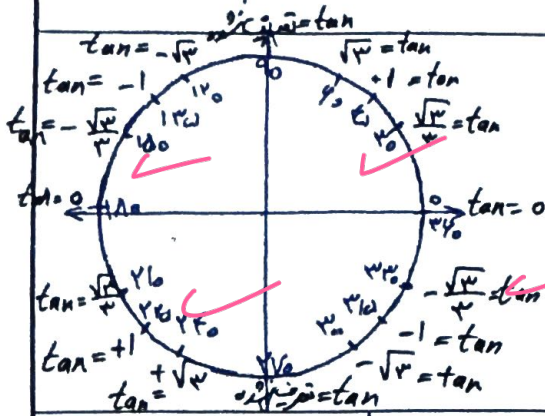




۲. اثبات یابی نوشتار

۲

الف:

$$\sin \frac{\pi}{4} + \cos \pi + \sqrt{2} \tan \frac{\pi}{4} = \frac{1}{\sqrt{2}} + (-1) + \sqrt{2} \cdot 1 = \frac{1}{\sqrt{2}} - 1 + \sqrt{2} = \frac{1}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{1 - \sqrt{2} + 2}{\sqrt{2}} = \frac{1 + \sqrt{2}}{\sqrt{2}}$$

ب:

$$-\cos \frac{\omega \pi}{9} + \sin \frac{\pi}{3} - \sqrt{2} \sin \frac{\pi}{3} = -(-\frac{\sqrt{2}}{\sqrt{2}}) + \frac{\sqrt{2}}{\sqrt{2}} - \sqrt{2} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}} - \frac{2}{\sqrt{2}} = \frac{\sqrt{2} + \sqrt{2} - 2}{\sqrt{2}} = \frac{2\sqrt{2} - 2}{\sqrt{2}} = \frac{2(\sqrt{2} - 1)}{\sqrt{2}}$$

ج:

$$\cot \frac{\pi}{3} + \sqrt{2} \cot \frac{\pi}{4} - \sqrt{2} \tan \frac{\pi}{4} = \frac{1}{\sqrt{2}} + \sqrt{2} \cdot 1 - \sqrt{2} \cdot 1 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

د:

$$\cos(\sin \pi) + \sin(\pi \cos \frac{\pi}{4}) = \cos(0) + \sin(\pi \cdot \frac{1}{\sqrt{2}}) = 1 + 1 = 2$$

$$f(x) = \begin{cases} x^2 + x & x \geq a \\ 2x + a + 2 & x \leq a \end{cases} \xrightarrow{x=a} a^2 + a = 2a + a + 2 \rightarrow a^2 + a - 2 = 0 \rightarrow (a+2)(a-1) = 0 \Rightarrow \begin{cases} a=1 \\ a=-2 \end{cases}$$

از آنجا که $a > 0$ پس $a=1$ می‌باشد. $a=-2$ غیر قابل قبول است.

$$f(1) \Rightarrow 1^2 + 1 = 2$$

$$f(x) = \begin{cases} 2x^2 - b & |x+1| \geq 2 \\ a + 2x & -2 \leq x \leq -1 \end{cases} \xrightarrow{x=-3} 18 - b = a - 4 \Rightarrow 18 + 4 = a + b \Rightarrow 22 = a + b$$

$$|x+1| \geq 2 \Rightarrow x+1 \geq 2 \Rightarrow x \geq 1$$

$$x+1 \leq -2 \Rightarrow x \leq -3$$

از آنجا که $x \geq 1$ یا $x \leq -3$ می‌باشد پس $x=1$ یا $x=-3$ در دو محله داده شده برای $f(x)$ عدد -3 را می‌توانیم در نظر بگیریم.

الف: $f = \frac{1}{x^3 - 4x^2}$

$$x^3 - 4x^2 \neq 0 \Rightarrow x^2(x-4) \neq 0$$

○ ۴

$$D_f = \mathbb{R} - \{0, 4\}$$

ب: $f = \frac{1}{2x^3 + 4x^2 + 2x + 4}$

$$2x^3 + 4x^2 + 2x + 4 \neq 0 \quad D_f = \mathbb{R} - \{-2\}$$

$$2x(x^2 + 2x + 1) + 4 \neq 0$$

$$(x^2 + 2x + 1)(2x + 4) \neq 0$$

$$x^2 + 2x + 1 = (x+1)^2 \neq 0 \Rightarrow x \neq -1$$

$$2x + 4 = 0 \Rightarrow x = -2$$

الف:

$$\sqrt{x^2-1} + \sqrt{x-2} =$$

$$x^2-1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow \begin{cases} x \geq 1 \\ x \leq -1 \end{cases}$$

$$x-2 \geq 0 \rightarrow \boxed{x \geq 2}$$



$$D_f = [2, +\infty)$$

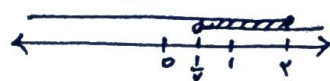
ب:

$$\frac{\sqrt{4-3x}}{\sqrt{4x-3}} =$$

$$4-3x \geq 0 \rightarrow 4 \geq 3x \rightarrow \boxed{x \leq \frac{4}{3}}$$

$$4x-3 > 0 \rightarrow 4x > 3 \rightarrow \boxed{x > \frac{3}{4}}$$

$$D_f = (\frac{3}{4}, \frac{4}{3}]$$



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$$f = \sqrt{4x-a} + \sqrt{b-3x}$$

$$D_f = \{x\}$$

$$\frac{b}{a} - \frac{4}{12} = \frac{1}{x}$$

$$4x-a \geq 0$$

$$4x \geq a$$

$$\boxed{x \geq \frac{a}{4}}$$

$$b-3x \geq 0$$

$$3x \leq b$$

$$\boxed{x \leq \frac{b}{3}}$$

دسته {x} است پس اشتراک دو مجموعه باید عدد ۲ باشد تا دامنه {x} به دست آید.



$$\frac{a}{4} = 2 \rightarrow \boxed{a = 8}$$

$$\frac{b}{3} = 2 \rightarrow \boxed{b = 6}$$

7

الف:

$$\sqrt{\frac{x^2-x-4}{x^2-13x^2+44}} \rightarrow \frac{(x-4)(x+1)}{(x-4)(x+1)(x-1)(x-4)} \rightarrow \frac{1}{(x-1)(x-4)}$$

$$x^2=t \Rightarrow t^2-13t+44 \Rightarrow (t-4)(t-11) \Rightarrow \boxed{t=4, 11}$$

$$(x^2-4)(x^2-11) \Rightarrow (x-2)(x+2)(x-\sqrt{11})(x+\sqrt{11})$$

$$D_f = (-\infty, -\sqrt{11}) \cup (-2, 2) \cup (2, +\infty)$$

ب:

$$\sqrt{\frac{x^3-2x^2-4x+4}{x^3+13x^2-44x-4}} \Rightarrow \frac{(x-1)(x-4)(x+1)}{(x+1)(x+4)(x-1)(x-4)} \geq 0$$

$$\frac{-3}{+8} \frac{-2}{-1} \frac{-1}{+1} \frac{1}{-1} \frac{2}{+1} \frac{3}{+1}$$

$$D_f = (-\infty, -2) \cup (-2, -1) \cup (-1, 2) \cup (2, +\infty)$$

8

الف:

$$\sqrt{2[x]-4}$$

$$2[x]-4 \geq 0$$

$$2[x] \geq 4 \rightarrow \boxed{[x] \geq 2}$$

$$D_f = [2, +\infty)$$

ب:

$$\sqrt{2[x]-4}$$

$$2[x]-4 \geq 0$$

$$2[x] \geq 4 \rightarrow \boxed{[x] \geq \frac{4}{2} = 2}$$

$$D_f = [2, +\infty)$$

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الف:

$$\sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{x^2+2x+1}} \rightarrow \Delta < 0 \rightarrow \text{...}$$

$$\frac{-1}{-1} \frac{3}{+1}$$

$$\frac{4}{+1} \frac{-1}{-1}$$

$$[-1, 3) \cup (3, +\infty) \cup [(-\infty, 4) \cup (4, 4)] =$$

$$D_f = [-1, 3) \cup (3, 4]$$

ب:

$$\sqrt{[x]-4} + \sqrt{2-[x]}$$

$$[x]-4 \geq 0$$

$$\boxed{[x] \geq 4}$$

$$2-[x] \geq 0$$

$$\boxed{[x] \leq 2}$$

از آنجا که بازه اشتراک به دست می آید دامنه می دارد.

$$D_f = \emptyset$$

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