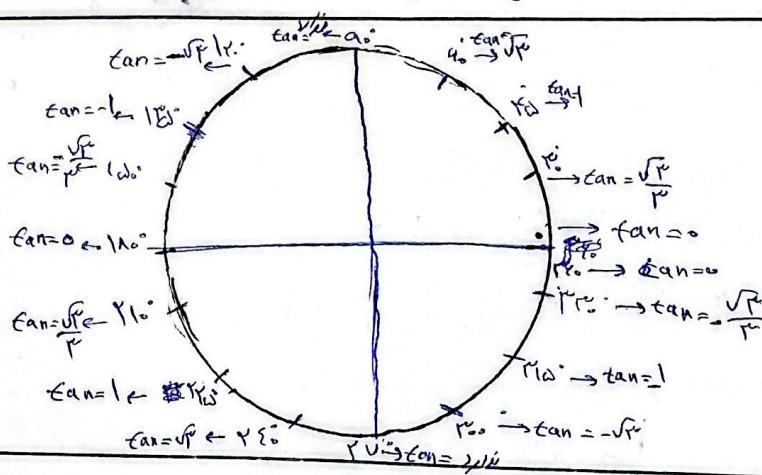




1- $x^2 + 2x = 2x + a + 2$ $\xrightarrow{x=a}$ $a^2 + 2a = 2a + a + 2 \rightarrow a^2 + a - 2 = 0$

$a^2 + a - 2 = 0$ $\xrightarrow{a+b+c=0}$ $\begin{cases} 1 \\ -2 \end{cases} \rightarrow a=1$ $F(x) = \begin{cases} x+2 & x \geq 1 \\ 2x+\frac{1}{x} & x < 1 \end{cases}$

$F(1) = 1+2 = 3$

* برای سوال 3 و 4 باید و یکبار است.

الف) $\sin \frac{x}{4} + \cos x + \tan \frac{3x}{2} = \sin 45^\circ + \cos 90^\circ + \tan 135^\circ = \frac{1}{\sqrt{2}} + 0 + (-1) = \frac{1}{\sqrt{2}} - 1$

ب) $-\cos \frac{\omega x}{4} + \sin \frac{2x}{\sqrt{2}} - \sin \frac{4x}{\sqrt{2}} = -\cos 135^\circ + \sin 90^\circ - \sin 180^\circ = -(-\frac{1}{\sqrt{2}}) + 1 - 0 = \frac{1}{\sqrt{2}} + 1$

ج) $\cot \frac{2x}{\sqrt{2}} + \cot \frac{4x}{\sqrt{2}} - \cot \frac{5x}{4} = \cot 90^\circ + \cot 180^\circ - \cot 112.5^\circ = 0 + 0 - \cot 112.5^\circ = -\cot 112.5^\circ$

د) $\cos(\sin x) + \sin(x \cos \frac{\pi}{4}) = \cos 0^\circ + \sin(\frac{\pi}{2}) = 1 + 1 = 2$

2- $f(x) = \begin{cases} 2x^2 - b & |x+1| \geq 2 \\ a+2x & -3 \leq x \leq -1 \end{cases} \rightarrow |x+1| \geq 2 \rightarrow x+1 \geq 2 \cup x+1 \leq -2 \rightarrow x \geq 1 \cup x \leq -3$

$2x^2 - b = a + 2x$ $\xrightarrow{x=-3}$ $18 - b = a + 2(-3) \rightarrow 18 - b = a - 6 \rightarrow a = 24 - b$

الف) $y = \frac{1}{x^3 - 2x^2}$

$x^3 - 2x^2 \neq 0 \rightarrow x^2(x-2) \neq 0 \rightarrow x \neq 0 \text{ و } x \neq 2$

$D_f = \mathbb{R} - \{0, 2\}$

ب) $\frac{1}{2x^3 + 5x^2 + 2x + 2}$

$2x^3 + 5x^2 + 2x + 2 \neq 0 \rightarrow 2x^2(x+2) + 2(x+1) \neq 0 \rightarrow (x+2)(2x^2+2) \neq 0 \rightarrow x \neq -2$

$D_f = \mathbb{R} - \{-2\}$

نقطه $x = -2$ از مخرج صفر می شود

الف) $y = \sqrt{x^2-1} + \sqrt{x-2}$ $I) x^2-1 \geq 0 \rightarrow (x-1)(x+1) \geq 0$ $II) x-2 \geq 0 \rightarrow x \geq 2$

$I \cap II = x \geq 2 = [2, +\infty)$

Sign chart for x^2-1 : x | -1 | 1 | $+$ | $-$ | $+$

$(-\infty, -1] \cup [1, +\infty)$

ب) $\frac{\sqrt{4-x}}{\sqrt{4x-2}}$ $I) 4x-2 > 0 \rightarrow 4x > 2 \rightarrow x > \frac{1}{2}$

$II) 4-x \geq 0 \rightarrow 4 \geq x \rightarrow x \leq 4$

$\frac{1}{2} < x \leq 4 \rightarrow D_f = (\frac{1}{2}, 4]$

$y = \sqrt{4x-a} + \sqrt{b-2x}$ $D_y = \{x\}$

$I: 4x-a \geq 0 \rightarrow 4x \geq a \rightarrow x \geq \frac{a}{4}$

$II: b-2x \geq 0 \rightarrow b \geq 2x \rightarrow \frac{b}{2} \geq x$

جواب $\frac{a}{4} = 2 \rightarrow a=8$ و $\frac{b}{2} = 2 \rightarrow b=4$

$\frac{b}{a} = \frac{4}{8} = \frac{1}{2}$

الف) $\sqrt{\frac{x^2-x-4}{x^2-12x^2+34}}$ $I: x^2-12x^2+34 > 0 \rightarrow -11x^2+34 > 0 \rightarrow x^2 < \frac{34}{11}$

$\Delta = b^2-4ac \rightarrow 144 - (4 \times 1 \times 34) = 20$

$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-12 \pm \sqrt{20}}{2 \times (-11)} = \frac{6 \pm \sqrt{5}}{11}$

$II: x^2-x-4 = 0 \rightarrow x = \frac{1 \pm \sqrt{17}}{2}$

جواب $(-\infty, -\frac{6+\sqrt{5}}{11}) \cup (\frac{6-\sqrt{5}}{11}, \frac{6+\sqrt{5}}{11}) \cup (\frac{1+\sqrt{17}}{2}, +\infty)$

الف) $\sqrt{2[x]-5}$ $2[x]-5 \geq 0 \rightarrow 2[x] \geq 5 \rightarrow [x] \geq \frac{5}{2} \rightarrow D_y = [\frac{5}{2}, +\infty)$

ب) $y = \sqrt{2[x]-a}$ $2[x]-a \geq 0 \rightarrow 2[x] \geq a \rightarrow [x] \geq \frac{a}{2} \rightarrow D_y = [\frac{a}{2}, +\infty)$

الف) $\sqrt{\frac{x+1}{|x-2|}} + \sqrt{\frac{4-x}{x^2+2x+5}}$ $D_f = I \cap II = [-1, 2) \cup (5, +\infty) \cap (-\infty, 4] = [-1, 2) \cup (5, 4]$

$I: \frac{x+1}{|x-2|} \geq 0$ $II: \frac{4-x}{x^2+2x+5} \geq 0$

$D_f = I \cap II = [5, +\infty) \cap (-\infty, 4] = \emptyset$