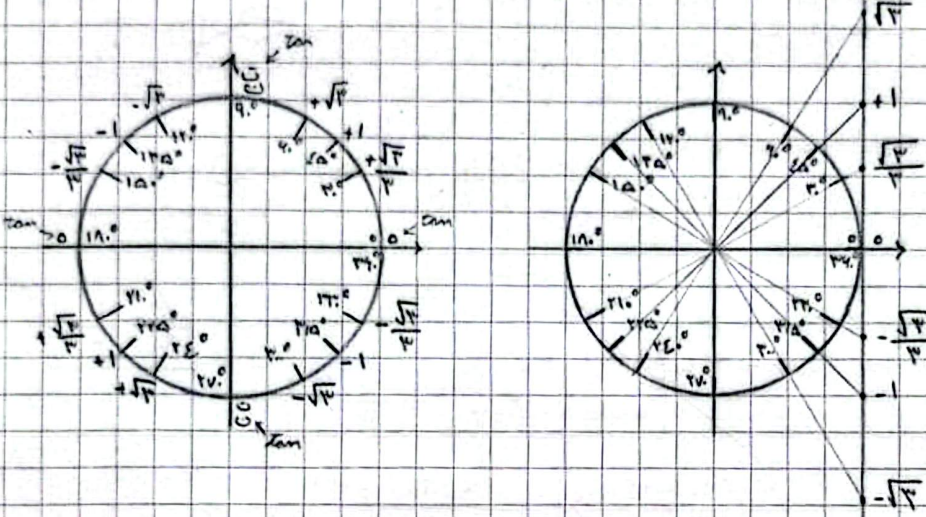


۱- چون مثلث متشابه در مثل منظره سواله بالاییه
فقط بنویسیم، جنسش کمیده.



الف) $\sin \frac{\pi}{4} + \cos \pi + 2 \tan \frac{3\pi}{4} = \frac{1}{2} + (-1) + 2(-1) = -\frac{3}{2}$

ب) $-\cos \frac{5\pi}{4} + \sin \frac{2\pi}{3} - 3 \sin \frac{\pi}{6} = -(-\frac{\sqrt{2}}{2}) + \frac{\sqrt{3}}{2} - 3(\frac{1}{2}) = \frac{\sqrt{2} + \sqrt{3} - 3}{2}$

ج) $\cot \frac{2\pi}{3} + 3 \cot \frac{\pi}{6} - 2 \tan \frac{5\pi}{4} = -\frac{\sqrt{3}}{3} + 3(\frac{\sqrt{3}}{3}) - 2(-\frac{\sqrt{2}}{2}) = \frac{\sqrt{3} + 2\sqrt{2}}{3}$

د) $\cos(\sin \pi) + \sin(\pi \cos \frac{\pi}{2}) = \cos(0) + \sin(\pi \cdot \frac{1}{2}) = \cos(0) + \sin(\frac{\pi}{2}) = 1 + 1 = 2$

$$f(x) = \begin{cases} x^2 + \varepsilon x & x \geq a \\ 2x + a + \varepsilon & x \leq a \end{cases}$$

(a > 0)

$$\begin{aligned} a^2 + \varepsilon a &= 2a + a + \varepsilon \rightarrow a^2 + a - \varepsilon = 0 \\ \rightarrow (a + \varepsilon)(a - 1) &= 0 \rightarrow a = -\varepsilon, 1 \end{aligned}$$

$\rightarrow a = 1$

$$\begin{aligned} f(1) &= 2(1) + 1 + \varepsilon = a \\ \varepsilon(1)^2 + \varepsilon(1) &= a \end{aligned}$$

$f(1) = a$

$$f(x) = \begin{cases} 2x^2 - b & |x+1| \geq 2 \\ a + 2x & -\frac{3}{2} \leq x \leq -1 \end{cases}$$

$$\begin{aligned} |x+1| \geq 2 &\rightarrow x+1 \geq 2, x+1 \leq -2 \rightarrow x \geq 1, x \leq -3 \\ 2(-\frac{3}{2})^2 - b &= a + 2(-1) \rightarrow 1 - b = a - 2 \rightarrow a + b = 3 \end{aligned}$$

$a + b = 3$

الف) $y = \frac{1}{x^2 - \varepsilon x^2}$

$$x^2 - \varepsilon x^2 \neq 0 \rightarrow x^2(x - \varepsilon) = 0 \rightarrow x^2 = 0 \rightarrow x = 0, x - \varepsilon = 0 \rightarrow x = \varepsilon$$

$D = \mathbb{R} - \{0, \varepsilon\}$

ب) $y = \frac{1}{2x^3 + \varepsilon x^2 + 2x + \varepsilon}$

$$\begin{aligned} 2x^3 + \varepsilon x^2 + 2x + \varepsilon &\neq 0 \rightarrow 2(x^3 + \frac{\varepsilon}{2}x^2 + x + \frac{\varepsilon}{2}) \neq 0 \rightarrow x^3 + \frac{\varepsilon}{2}x^2 + x + \frac{\varepsilon}{2} \neq 0 \rightarrow (x + \frac{\varepsilon}{2})(x^2 + 1) = 0 \\ \rightarrow x + \frac{\varepsilon}{2} &= 0 \rightarrow x = -\frac{\varepsilon}{2}, x^2 + 1 = 0 \rightarrow x^2 = -1 \end{aligned}$$

$D = \mathbb{R} - \{-\frac{\varepsilon}{2}\}$

s.a.m

$$\text{الف) } y = \sqrt{x^2 - 1} + \sqrt{x - 2}$$

$$\begin{aligned} x^2 - 1 \geq 0 &\rightarrow x^2 \geq 1 \rightarrow x \geq 1 \vee x \leq -1 \quad (I) \\ x - 2 \geq 0 &\rightarrow x \geq 2 \quad (II) \end{aligned}$$

$$I \cap II \rightarrow \mathcal{D} = [2, +\infty)$$

$$\text{ب) } y = \frac{\sqrt{4 - 3x}}{\sqrt{4x - 3}}$$

$$\begin{aligned} 4 - 3x \geq 0 &\rightarrow 3x \leq 4 \rightarrow x \leq \frac{4}{3} \quad (I) \\ 4x - 3 \geq 0 &\rightarrow 4x \geq 3 \rightarrow x \geq \frac{3}{4} \quad (II) \end{aligned}$$

$$I \cap II \rightarrow \frac{3}{4} \leq x \leq \frac{4}{3} \rightarrow \mathcal{D} = \left[\frac{3}{4}, \frac{4}{3}\right]$$

$$y = \sqrt{4x - a} + \sqrt{b - 3x}$$

$$\begin{aligned} 4x - a \geq 0 &\rightarrow 4x \geq a \rightarrow x \geq \frac{a}{4} \\ b - 3x \geq 0 &\rightarrow 3x \leq b \rightarrow x \leq \frac{b}{3} \end{aligned}$$

$$\frac{b}{a} = \frac{1}{2}$$

$$\mathcal{D} = \{1\}$$

بما أن $\sqrt{4x-a} + \sqrt{b-3x}$ معرّف فقط إذا كان $4x-a \geq 0$ و $b-3x \geq 0$ أي $x \geq \frac{a}{4}$ و $x \leq \frac{b}{3}$ أي $\frac{a}{4} \leq x \leq \frac{b}{3}$ و بما أن $\frac{b}{a} = \frac{1}{2}$ أي $b = \frac{a}{2}$ إذن $\frac{a}{4} \leq x \leq \frac{\frac{a}{2}}{3} = \frac{a}{6}$ و بما أن $\frac{a}{4} \leq \frac{a}{6}$ فقط إذا كان $a = 0$ و $b = 0$ و في هذه الحالة $\mathcal{D} = \{1\}$

$$\mathcal{D} = \left[\frac{a}{4}, \frac{b}{3}\right] \rightarrow \frac{a}{4} = \frac{b}{3} = 1 \rightarrow a = 4, b = 3 \rightarrow \frac{b}{a} = \frac{3}{4} \neq \frac{1}{2}$$

$$\text{الف) } y = \sqrt{\frac{x^2 - x - 4}{x^2 - 13x^2 + 14}}$$

$$\frac{x^2 - x - 4}{x^2 - 13x^2 + 14} \geq 0 \rightarrow \frac{(x-3)(x+1)}{(x-9)(x-2)} \geq 0 \rightarrow \frac{(x-3)(x+1)}{(x-9)(x-2)(x-9)(x-2)} \geq 0$$

$$+ \frac{+}{-} - \frac{+}{-} - \frac{+}{-} + \frac{+}{-} +$$

$$\mathcal{D} = (-\infty, 3) \cup (9, 2) \cup (2, +\infty)$$

$$= (-\infty, 3) \cup (2, +\infty) - \{9\}$$

$$\text{ب) } y = \sqrt{\frac{x^2 - 2x^2 - 5x + 4}{x^2 + 2x^2 - 5x - 4}}$$

$$\frac{x^2 - 2x^2 - 5x + 4}{x^2 + 2x^2 - 5x - 4} \geq 0 \rightarrow \frac{(x-1)(x-3)(x+1)}{(x-2)(x+1)(x+4)} \geq 0$$

$$+ \frac{+}{-} - \frac{+}{-} + \frac{+}{-} - \frac{+}{-} + \frac{+}{-} +$$

$$\mathcal{D} = (-\infty, 3) \cup [-2, -1) \cup [1, 2) \cup [4, +\infty)$$

$$\text{الف) } y = \sqrt{2[x] - 5}$$

$$2[x] - 5 \geq 0 \rightarrow 2[x] \geq 5 \rightarrow [x] \geq \frac{5}{2} \rightarrow x \geq 2 \rightarrow \mathcal{D} = [2, +\infty)$$

$$\text{ب) } y = \sqrt{2[x] - 5}$$

$$2[x] - 5 \geq 0 \rightarrow 2[x] \geq 5 \rightarrow [x] \geq \frac{5}{2} \rightarrow x \geq 2 \rightarrow \mathcal{D} = [2, +\infty)$$

$$\text{الف) } y = \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{x^2+2x+5}}$$

$$\frac{x+1}{|x-3|} \geq 0 \rightarrow x+1 \geq 0 \rightarrow x \geq -1 \wedge \frac{4-x}{x^2+2x+5} \geq 0 \rightarrow 4-x \geq 0 \rightarrow x \leq 4 \rightarrow [-1, 4] \cup (3, +\infty) \quad (I)$$

$$\frac{4-x}{x^2+2x+5} \geq 0 \rightarrow 4-x \geq 0 \rightarrow x \leq 4 \rightarrow \frac{4-x}{x^2+2x+5} = \frac{4-x}{(x+1)^2+4} \geq 0 \rightarrow (-\infty, 4] \quad (II)$$

$I \cap II$

$$\mathcal{D} = [-1, 3) \cup (3, 4]$$

$$\text{ب) } y = \sqrt{[x] - 5} + \sqrt{2 - [x]}$$

$$\begin{aligned} [x] - 5 \geq 0 &\rightarrow [x] \geq 5 \\ 2 - [x] \geq 0 &\rightarrow [x] \leq 2 \end{aligned}$$

$$\mathcal{D} = \emptyset$$

ملاحظة: لا يوجد عدد صحيح x يحقق $5 \leq [x] \leq 2$ و هذا هو الجواب النهائي.