

۲. آزمون پایانی و نامل فرست

الف) $P(x) = 2x^3 + 2x^2 - 11x + 4 \Rightarrow$ مجزیه ضرایب $\Rightarrow 2 + 4 - 11 + 4 = 0$

$(x-1)(2x^2 + 4x - 4) \quad (2x-1)(x+4)$

$P(x) = (x-1)(2x-1)(x+4) \Rightarrow$ تجزیه $\neq 0 \Rightarrow x \neq 1, x \neq \frac{1}{2}, x \neq -4$

$D_f = \mathbb{R} - \left\{ -4, \frac{1}{2}, 1 \right\}$

ب) $P(x) = 2x^3 + 9x^2 + 10x + 4 \Rightarrow$ ضرایب توان زوج = ضرایب توان فرد $\Rightarrow 4 + 10 = 14, 2 + 9 = 11$

$\Rightarrow (x+1)(2x^2 + 7x + 4) \Rightarrow (2x+1)(x+4)$

$P(x) = (x+1)(2x+1)(x+4) \Rightarrow$ تجزیه $\neq 0 \Rightarrow x \neq -1, x \neq -\frac{1}{2}, x \neq -4 \Rightarrow D_f = \mathbb{R} - \left\{ -4, -\frac{1}{2}, -1 \right\}$

الف) $P(x) = x^3 - 2x^2 + 2x - 1 \Rightarrow$ مجزیه ضرایب $1 - 2 + 2 - 1 = 0 \Rightarrow (x-1)(x^2 - x + 1)$ $(b^2 - 4ac) = (-1)^2 - 4 = -3$

$D_f = \mathbb{R} - \{1\}$

$x \neq 1$

$\Delta < 0 \Rightarrow$ ضرایب زوج $\neq$ ضرایب فرد

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ب) $x^3 - 2x^2 + 2x - 1 \Rightarrow$ تجزیه ضرایب $1 - 2 + 2 - 1 = 0 \Rightarrow (x-1)(x^2 - x + 1) \Rightarrow x \neq 1$

$\frac{x+3}{x-1} \geq 0 \Rightarrow$ $D_f = (-\infty, -3] \cup (1, +\infty)$

$y = \frac{2}{x^2 - 5|x-1| - 2x + 5} \Rightarrow$ تجزیه $x^2 - 2x + 5 \Rightarrow (x-1)^2 + 4 \Rightarrow (x-1)^2 = |x-1|^2$

$|x-1|^2 + 4 - 5|x-1| = 0 \Rightarrow (t-1)(t-4) = 0 \Rightarrow t=1 \quad t=4$

$|x-1|=t \Rightarrow$ ① $|x-1|=1 \Rightarrow x=-1=1-2 \Rightarrow x-1=-1 \Rightarrow x=0$

$\Rightarrow$ ② $|x-1|=4 \Rightarrow x=5 \quad \vee \quad x=-3 \Rightarrow x \in \{0, 2, 5, -4\}$

$D_f = \mathbb{R} - \{-4, 0, 2, 5\}$

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الف) $y = \sqrt{-x} - 2 \Rightarrow [-x] - 2 \geq 0 \Rightarrow [-x] \geq 2$

$\Rightarrow x \in \mathbb{Z} \Rightarrow \lfloor x \rfloor \geq 2 \Rightarrow -x \geq 2 \Rightarrow x \leq -2 \Rightarrow D_f = (-\infty, -2]$

ب) $\lfloor x^2 \rfloor - 2\lfloor x \rfloor - 4 \neq 0 \Rightarrow (\lfloor x \rfloor - 4)(\lfloor x \rfloor + 1) \neq 0$

① $\lfloor x \rfloor - 4 \neq 0 \Rightarrow \lfloor x \rfloor \neq 4 \Rightarrow x \notin [4, 5)$

② $\lfloor x \rfloor + 1 \neq 0 \Rightarrow \lfloor x \rfloor \neq -1 \Rightarrow x \notin [-1, 0)$

$D_f = \mathbb{R} - ([-1, 0) \cup [4, 5))$

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الف) $\lfloor x - \frac{1}{4} \rfloor \geq 0 \Rightarrow \lfloor A \rfloor \geq 0 \Rightarrow A \geq 0 \Rightarrow x - \frac{1}{4} \geq 0 \Rightarrow x \geq \frac{1}{4} \Rightarrow D_f = [\frac{1}{4}, +\infty)$

ب) $\lfloor x - \frac{1}{4} \rfloor + \lfloor \frac{1}{4} - x \rfloor \geq 0 \Rightarrow \lfloor u \rfloor + \lfloor -u \rfloor = -1 \Rightarrow u \in \mathbb{Z} \Rightarrow$ ضرایب زوج $\neq$ ضرایب فرد $\Rightarrow \lfloor u \rfloor + \lfloor -u \rfloor = 0$ $\checkmark$

$u \notin \mathbb{Z} \Rightarrow$ ضرایب زوج = ضرایب فرد $\Rightarrow u \in \mathbb{Z}$

$x - \frac{1}{4} \in \mathbb{Z} \Rightarrow x - \frac{1}{4} = k \Rightarrow (k \in \mathbb{Z}) \Rightarrow x = k + \frac{1}{4}$

$D_f = \left\{ x \in \mathbb{R} \mid x = k + \frac{1}{4}, k \in \mathbb{Z} \right\}$

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r, q, p

الف) $\tan x + 1 \neq 0 \Rightarrow \tan x \neq -1 \Rightarrow x \neq \frac{3\pi}{4} + k\pi$ ① $x = \frac{\pi}{4} + k\pi \Rightarrow \tan x = 1$
 $D_f = \mathbb{R} - \left\{ k\pi, \frac{\pi}{4} + k\pi, \frac{3\pi}{4} + k\pi \mid k \in \mathbb{Z} \right\}$ ② $x = k\pi \Rightarrow \tan x = 0$

ب) $1 - \sin^2 x \geq 0 \Rightarrow \sin^2 x \leq 1 \Rightarrow \sin x \leq \frac{1}{\sqrt{2}} \Rightarrow -\frac{1}{\sqrt{2}} \leq \sin x \leq \frac{1}{\sqrt{2}} \Rightarrow$
 $-\frac{\pi}{4} + k\pi \leq x \leq \frac{\pi}{4} + k\pi$
 $D_f = \left[-\frac{\pi}{4} + k\pi, \frac{\pi}{4} + k\pi \right] \rightarrow k \in \mathbb{Z}$

ج) $1 - \sin x \geq 0 \Rightarrow \sin x \leq 1 \Rightarrow \frac{\pi}{4} + 2k\pi \leq x \leq \frac{5\pi}{4} + 2k\pi$
 $D_f = \left[\frac{\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi \right] \rightarrow k \in \mathbb{Z}$
 د) $1 - \cos x \geq 0 \Rightarrow \cos x \leq 1 \Rightarrow \frac{\pi}{2} + 2k\pi \leq x \leq \frac{3\pi}{2} + 2k\pi$
 $D_f = \left[\frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi \right] \rightarrow k \in \mathbb{Z}$

الف) $D_f \arcsin \rightarrow [-1, 1]$
 $\Rightarrow -1 \leq x^2 + 2x + 1 \leq 1$
 ① $x^2 + 2x + 1 \geq -1 \Rightarrow x^2 + 2x + 2 \geq 0 \Rightarrow (x+1)^2 \geq 0 \Rightarrow x \geq -1$
 ② $x^2 + 2x + 1 \leq 1 \Rightarrow x^2 + 2x \leq 0 \Rightarrow x(x+2) \leq 0 \Rightarrow -2 \leq x \leq 0$
 ① $\cap$ ② $\Rightarrow [-2, 0] \cup [-1, 1] \Rightarrow D_f = [-2, -1] \cup [-1, 0]$

الف) $y = \log_{\frac{1}{2}}(1 - \log_2 x)$ ① $x > 0$ ② $1 - \log_2 x > 0 \Rightarrow \log_2 x < 1 \Rightarrow \log_2 x < \log_2 2 \Rightarrow x < 2$
 $D_f = (0, 2)$
 ب) $\log_{\frac{1}{2}}(1 - \log_{\frac{1}{2}} x) \Rightarrow$ ① $x > 0$ ② $1 - \log_{\frac{1}{2}} x > 0 \Rightarrow \log_{\frac{1}{2}} x < 1 \Rightarrow \log_{\frac{1}{2}} x < \log_{\frac{1}{2}} \frac{1}{2} \Rightarrow x > \frac{1}{2}$
 $D_f = (\frac{1}{2}, +\infty)$

الف) $r(x - x^r - 1) \geq 0 \Rightarrow r(x - x^r) \geq 1 \Rightarrow x = r^{\frac{1}{r}} \Rightarrow r(x - x^r) \geq r^{\frac{1}{r}}$
 $\Rightarrow (x - x^r) \geq \frac{1}{r} \Rightarrow x^r - rx + 1 \leq 0 \Rightarrow (x-1)(x-r) \leq 0$
 $D_f = [1, r]$
 ب) $\frac{rx+d}{rx+c} \geq 0 \Rightarrow$ ① $r > 0$ ② $r < 0$
 $\Rightarrow \frac{rx+d}{rx+c} = k \ (k \in \mathbb{Z}) \Rightarrow rx + d = r k x + c k \Rightarrow rx - r k x = c k - d$

$D_f = \left\{ -\frac{d}{r}, 1, r, \dots \right\} \leftarrow D_f = \left\{ x \mid x = \frac{ck-d}{r-rk}, k \in \mathbb{Z} \right\}$

$$\cos y = \sqrt{a - rx}$$

سوال (ب) :

$$\textcircled{1} a - rx \geq 0 \Rightarrow rx \leq a \Rightarrow x \leq \frac{a}{r} \quad \textcircled{2} -1 \leq \cos x \leq 1 \Rightarrow -1 \leq \sqrt{a - rx} \leq 1$$

$$\Rightarrow \sqrt{a - rx} \geq 0 \rightarrow \sqrt{a - rx} \leq 1 \rightarrow a - rx \leq 1 \rightarrow -rx \leq -a + 1 \rightarrow x \geq \frac{a - 1}{r}$$

$$\Rightarrow \left(x \leq \frac{a}{r} \cap x \geq \frac{a - 1}{r} \right) \rightarrow D_f = \left[\frac{a - 1}{r}, \frac{a}{r} \right]$$