

(۱) $y = \frac{x+3}{2x^3+3x^2-11x+10} \Rightarrow \frac{2x^3+3x^2-11x+10}{-2x^3+3x^2-11x+10} \mid \frac{x-1}{2x^3+3x^2-11x+10}$
 $\Rightarrow 2x^3+3x^2-11x+10 = 0 \Rightarrow x^2+3x-4 = 0 \Rightarrow (x-1)(x+4) = 0 \Rightarrow x = \frac{1}{2}, -4 \Rightarrow x = 1$
 $\Rightarrow D_y = \mathbb{R} - \left\{ -3, \frac{1}{2}, 1 \right\}$

(ب) $y = \frac{x+3}{2x^3+9x^2+10x+4} \Rightarrow \frac{2x^3+9x^2+10x+4}{-2x^3+9x^2+10x+4} \mid \frac{x+1}{2x^3+9x^2+10x+4}$
 $\Rightarrow 2x^3+9x^2+10x+4 = 0 \Rightarrow x^2+7x+4 = 0 \Rightarrow (x+1)(x+4) = 0 \Rightarrow x = -\frac{1}{4}, -3 \Rightarrow x = -1$
 $\Rightarrow D_y = \mathbb{R} - \left\{ -3, -1, -\frac{1}{4} \right\}$

(۲) (الف) $y = \frac{x+3}{x^3-2x^2+2x-1} \Rightarrow \frac{2x^3-2x^2+2x-1}{-2x^3+2x^2-2x+1} \mid \frac{x-1}{x^3-2x^2+2x-1}$
 $\Rightarrow x^3-2x^2+2x-1 = 0 \Rightarrow 1-2+2-1 = 0 \Rightarrow \Delta < 0$
 $\Rightarrow x = 1$
 $\Rightarrow D_y = \mathbb{R} - \{1\}$

(ب) $y = \sqrt{\frac{x+3}{x^3-2x^2+2x-1}} \Rightarrow \frac{x+3}{x^3-2x^2+2x-1} \geq 0 \Rightarrow \frac{x+3}{x^3-2x^2+2x-1} \geq 0 \Rightarrow \frac{x+3}{x^3-2x^2+2x-1} \geq 0$
 $\Rightarrow D_y = (-\infty, -3] \cup (1, +\infty)$

(۳) $y = \frac{1}{x^2-2|x-1|-11x+5} \Rightarrow x^2-2|x-1|-11x+5 \neq 0$
 $\Rightarrow x \leq 1 \Rightarrow x^2-2(x-1)-11x+5 = 0 \Rightarrow x^2-13x+7 = 0 \Rightarrow (x-2)(x-5) = 0 \Rightarrow x = 2, 5$
 $\Rightarrow x > 1 \Rightarrow x^2-2(1-x)-11x+5 = 0 \Rightarrow x^2-9x = 0 \Rightarrow x(x-9) = 0 \Rightarrow x = 0, 9$
 $\Rightarrow D_y = \mathbb{R} - \{-3, 0, 2, 5\}$

(۴) (الف) $y = \sqrt{[-x]-2} \Rightarrow [-x]-2 \geq 0 \Rightarrow [-x] \geq 2 \Rightarrow x \leq -2 \Rightarrow D_y = (-\infty, -2]$

(ب) $y = \frac{5x^2+4}{[x]^2-2[x]-3} \Rightarrow [x]^2-2[x]-3 \neq 0 \Rightarrow t^2-2t-3 = 0 \Rightarrow (t-3)(t+1) = 0 \Rightarrow t = 3, -1$
 $\Rightarrow [x] = 3 \Rightarrow 3 \leq x < 4 \Rightarrow [x] = -1 \Rightarrow -1 \leq x < 0 \Rightarrow D_y = \mathbb{R} - ([-1, 0) \cup [3, 4])$

(۵) (الف) $y = \sqrt{\left[x - \frac{1}{3}\right] + \left[x + \frac{1}{3}\right]} \Rightarrow \left[x - \frac{1}{3}\right] + \left[x + \frac{1}{3}\right] \geq 0 \Rightarrow \left[x + \frac{1}{3} - 1\right] + \left[x + \frac{1}{3}\right] \geq 0 \Rightarrow \left[x + \frac{1}{3}\right] - 1 + \left[x + \frac{1}{3}\right] \geq 0$
 $\Rightarrow 2\left[x + \frac{1}{3}\right] \geq 1 \Rightarrow \left[x + \frac{1}{3}\right] \geq \frac{1}{2} \Rightarrow x + \frac{1}{3} \geq 1 \Rightarrow x \geq \frac{2}{3} \Rightarrow D_y = \left[\frac{2}{3}, +\infty\right)$

(ب) $y = \sqrt{\left[x - \frac{1}{3}\right] + \left[\frac{1}{3} - x\right]} \Rightarrow y = \sqrt{\left[x - \frac{1}{3}\right] + \left[-(x - \frac{1}{3})\right]} \Rightarrow \begin{cases} a \in \mathbb{Z} & [a] + [-a] = 0 \\ a \notin \mathbb{Z} & [a] + [-a] = -1 \end{cases} \Rightarrow x - \frac{1}{3} \in \mathbb{Z}$
 $\Rightarrow D_y = \left\{ x \mid x = k + \frac{1}{3}, k \in \mathbb{Z} \right\}$

$$\text{الف) } y = \frac{\cot x + 1}{\tan x + 1} \rightarrow \cot x = \frac{\cos x}{\sin x} \rightarrow \sin x \neq 0 \rightarrow x \neq k\pi \quad (k \in \mathbb{Z})$$

$$\rightarrow \tan x = \frac{\sin x}{\cos x} \rightarrow \cos x \neq 0 \rightarrow x \neq \frac{\pi}{2} + k\pi \quad (k \in \mathbb{Z}) \Rightarrow \mathcal{D}_f = \mathbb{R} \setminus \left\{ k\pi, k\pi + \frac{\pi}{2}, k\pi + \frac{3\pi}{2} \right\}$$

$$\rightarrow \tan x + 1 \neq 0 \rightarrow \tan x \neq -1 \rightarrow x \neq \frac{3\pi}{4} + k\pi \quad (k \in \mathbb{Z})$$

$$\text{ب) } y = \sqrt{1 - \sin x} \rightarrow 1 - \sin x \geq 0 \rightarrow \sin x \leq 1 \rightarrow \sin x \leq \frac{1}{2} \rightarrow \sin x \leq \frac{1}{2} \rightarrow -\frac{1}{2} \leq \sin x \leq \frac{1}{2}$$

$$\Rightarrow \mathcal{D}_f = \left[k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4} \right]$$

$$\text{الف) } y = \sqrt{2 \sin x - 1} \rightarrow 2 \sin x - 1 \geq 0 \rightarrow 2 \sin x \geq 1 \rightarrow \sin x \geq \frac{1}{2} \rightarrow \mathcal{D}_f = \left[k\pi + \frac{\pi}{6}, k\pi + \frac{5\pi}{6} \right]$$

$$\text{ب) } y = \sqrt{1 - 2 \cos x} \rightarrow 1 - 2 \cos x \geq 0 \rightarrow 2 \cos x \leq 1 \rightarrow \cos x \leq \frac{1}{2} \rightarrow \mathcal{D}_f = \left[k\pi + \frac{\pi}{3}, k\pi + \frac{5\pi}{3} \right]$$

$$\text{الف) } y = \arcsin(x^2 + 3x + 1) \rightarrow -1 \leq x^2 + 3x + 1 \leq 1 \rightarrow x^2 + 3x + 1 \geq 0 \rightarrow (x+1)(x+2) \geq 0 \rightarrow \frac{-1}{+1-1+} \quad (1)$$

$$\Rightarrow x^2 + 3x \leq 0 \rightarrow x(x+3) \leq 0 \rightarrow \frac{-3}{+1-1+} \Rightarrow \mathcal{D}_f = [-3, -1] \cup [-1, 0]$$

$$\text{ب) } \cos y = \sqrt{\Delta - 2x} \rightarrow \Delta - 2x \geq 0 \rightarrow 2x \leq \Delta \rightarrow x \leq \frac{\Delta}{2} \quad (I)$$

$$\rightarrow y = \arccos \rightarrow \sqrt{\Delta - 2x} \leq 1 \rightarrow \Delta - 2x \leq 1 \rightarrow 2x \geq \Delta - 1 \rightarrow x \geq \frac{\Delta - 1}{2} \quad (II) \Rightarrow \mathcal{D}_f = \left[\frac{\Delta - 1}{2}, \frac{\Delta}{2} \right]$$

$$\text{الف) } y = \log_p(1 - \log_p x) \rightarrow 1 - \log_p x > 0 \rightarrow \log_p x < 1 \rightarrow x < p \quad (I)$$

$$\log_p x \rightarrow x > 0 \quad (II)$$

$$I \cap II \rightarrow 0 < x < p \rightarrow \mathcal{D}_f = (0, p)$$

$$\text{ب) } y = \log_p(1 - \log_p \frac{x}{p}) \rightarrow 1 - \log_p \frac{x}{p} > 0 \rightarrow \log_p \frac{x}{p} < 1 \rightarrow \frac{x}{p} < p \rightarrow x < p^2 \quad (I)$$

$$\log_p \frac{x}{p} \rightarrow \frac{x}{p} > 0 \rightarrow x > 0 \quad (II)$$

$$I \cap II \rightarrow x > \frac{1}{p} \rightarrow \mathcal{D}_f = \left(\frac{1}{p}, p^2 \right)$$

$$\text{الف) } y = \sqrt{1 - x^{2p} - x^p} \rightarrow 1 - x^{2p} - x^p \geq 0 \rightarrow x^{2p} + x^p \leq 1 \rightarrow x^p \leq 1 \rightarrow x \leq 1 \quad (I)$$

$$\rightarrow (x-1)(x-p) \leq 0 \rightarrow \frac{1}{+1-1+} \rightarrow \mathcal{D}_f = [1, p]$$

$$\text{ب) } y = \left(\frac{px + \Delta}{px + \varepsilon} \right)! \rightarrow \frac{px + \Delta}{px + \varepsilon} \in \mathbb{W} \rightarrow x = \frac{-\varepsilon w + \Delta}{p w - p} \rightarrow \mathcal{D}_f = \left\{ x \mid x = \frac{-\varepsilon k + \Delta}{p k - p}, k \in \mathbb{W} \right\}$$