

$$f(x) = \sqrt{(a-x)x^2 + ax + b} \quad D_f = [3, +\infty) \quad x=3 \rightarrow 9(a-3) + 3a + b = 0 \Rightarrow$$

$$9a - 18 + 3a + b = 0 \rightarrow 12a + b - 18 = 0 \rightarrow b = -12a + 18 \quad \Delta = 0 \rightarrow a^2 - 4b(a-3) = 0$$

$$\Rightarrow a^2 - 4(-12a + 18)(a-3) = 0 \Rightarrow (4a-12)^2 = 0 \rightarrow a = \frac{12}{4} = 3 \quad b = -12 \times \frac{12}{4} + 18 = -27 + 18 = -9$$

$$b = \frac{12^2 - 144}{4} \Rightarrow b = -\frac{12}{4}$$

$$f(x) = \sqrt{x^2 + 2x + 2 - m^2} \quad x^2 + 2x + 2 - m^2 \geq 0 \quad \Delta \leq 0 \rightarrow 4 - 4(2 - m^2) \leq 0 \rightarrow$$

$$4 - 8 + 4m^2 \leq 0 \rightarrow 4m^2 - 4 \leq 0 \rightarrow m^2 - 1 \leq 0 \rightarrow (m-1)(m+1) \leq 0 \rightarrow -1 \leq m \leq 1 \rightarrow 1 - (-1) = 2$$

$$f(x) = \frac{\sqrt{4-x^2}}{[x] + [-x] + 1} \quad I: 4-x^2 \geq 0 \rightarrow x^2 \leq 4 \rightarrow -2 \leq x \leq 2$$

$$II: [x] + [-x] + 1 \neq 0 \quad III: [x] + [-x] + 1 = 0 \rightarrow x \in \mathbb{Z}$$

$$I \cap II: [-2, 2] \rightarrow x \in \mathbb{Q}$$

$$الف) y = \sqrt{x^3 + 3x^2} \rightarrow x^3 + 3x^2 \geq 0 \rightarrow x^2(x+3) \geq 0 \rightarrow x \geq -3 \quad D_f = [-3, +\infty)$$

$$ب) y = \sqrt{\frac{(x^2-1)(x^3-3x^2)}{x^3-1}} \rightarrow \frac{(x^2-1)x^2(x-3)}{(x-1)(x^2+x+1)} \geq 0 \rightarrow \frac{-1}{+} \frac{+}{-} \frac{+}{-} \frac{+}{+} \quad D_f = (-\infty, -1] \cup \{0\} \cup [3, +\infty)$$

$$الف) y = \sqrt{|x-2|-3} \rightarrow |x-2|-3 \geq 0 \rightarrow |x-2| \geq 3 \rightarrow \begin{cases} x-2 \geq 3 \rightarrow x \geq 5 \\ x-2 \leq -3 \rightarrow x \leq -1 \end{cases} \quad D_f = (-\infty, -1] \cup [5, +\infty)$$

$$ب) y = \sqrt{4-|2-x|} \rightarrow 4-|2-x| \geq 0 \rightarrow |2-x| \leq 4 \rightarrow \begin{cases} 2-x \leq 4 \rightarrow x \geq -2 \\ 2-x \geq -4 \rightarrow x \leq 6 \end{cases} \quad D_f = [-2, 6]$$

$$الف) y = \sqrt{1-x|x|} \rightarrow 1-x|x| \geq 0 \rightarrow x \geq 0 \rightarrow 1-x^2 \geq 0 \rightarrow x^2 \leq 1 \rightarrow 0 \leq x \leq 1$$

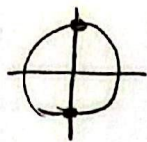
$$x < 0 \rightarrow 1-x(-x) \geq 0 \rightarrow 1+x^2 \geq 0 \rightarrow x < 0$$

$$D_f = (-\infty, -1]$$

$$ب) y = \frac{1}{\sqrt{3-|x-5|-1}} \rightarrow 3-|x-5|-1 \geq 0 \rightarrow 2-|x-5| \geq 0 \rightarrow |x-5| \leq 2 \rightarrow \begin{cases} x-5 \leq 2 \rightarrow x \leq 7 \\ x-5 \geq -2 \rightarrow x \geq 3 \end{cases}$$

$$D_f = (3, 7)$$

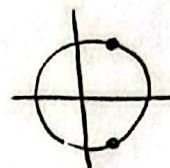
الف) $y = \frac{\sin^2 x - 1}{\cos x} \rightarrow \cos x \neq 0$



$D_f = \mathbb{R} - \{k\pi + \frac{\pi}{2}\}$

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ب) $y = \frac{\cos x}{r \cos x - 1} \rightarrow r \cos x - 1 \neq 0 \rightarrow \cos x \neq \frac{1}{r}$



$D_f = \mathbb{R} - \{k\pi \pm \frac{\pi}{r}\}$

الف) $y = \frac{x+r}{|rx+1| - |x+r|} \rightarrow |rx+1| - |x+r| \neq 0 \rightarrow |rx+1| \neq |x+r| \rightarrow$

$(rx+1)^2 \neq (x+r)^2 \rightarrow rx^2 + rx + 1 \neq x^2 + rx + r \rightarrow rx^2 - x - r \neq 0 \rightarrow x^2 - \frac{1}{r}x - r \neq 0$

$(x-\frac{1}{r})(x+r) \neq 0 \rightarrow x \neq \frac{1}{r}, x \neq -r \quad D_f = \mathbb{R} - \{-\frac{1}{r}, r\}$

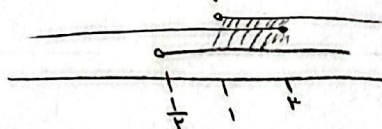
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ب) $y = \sqrt{|rx+1| - |x+r|} \rightarrow |rx+1| - |x+r| \geq 0 \rightarrow |rx+1| \geq |x+r| \rightarrow (rx+1)^2 \geq (x+r)^2$

$\rightarrow rx^2 + rx + 1 \geq x^2 + rx + r \rightarrow rx^2 - x - r \geq 0 \rightarrow \frac{-\frac{1}{r}}{r - \frac{1}{r}} \leq x \leq \frac{r}{r - \frac{1}{r}} \quad D_f = (-\infty, -\frac{1}{r}] \cup [\frac{r}{r - \frac{1}{r}}, +\infty)$

$f(x) = \sqrt{\log \log_a (rx-1)}$ I: $rx-1 > 0 \rightarrow rx > 1 \rightarrow x > \frac{1}{r}$ II: $\log \log_a (rx-1) \geq 0 \rightarrow \log_a (rx-1) \geq 1 \rightarrow rx-1 \geq a \rightarrow rx \geq a+1 \rightarrow x \geq \frac{a+1}{r}$

III: $\log_a (rx-1) < 0 \rightarrow rx-1 < 1 \rightarrow rx < 2 \rightarrow x < \frac{2}{r}$

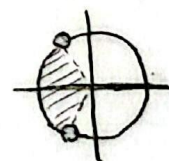


$D_f = (1, 2]$

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الف) $y = \log(r \cos x + 1)$ $r \cos x + 1 > 0 \rightarrow r \cos x > -1 \rightarrow \cos x > -\frac{1}{r}$

$D_f = (k\pi - \frac{\pi}{r}, k\pi + \frac{\pi}{r})$



ب) $y = \sqrt{\log \frac{x-1}{x+1}}$ I: $\frac{x-1}{x+1} > 0 \rightarrow \frac{-1}{x+1} > 0 \rightarrow x < -1$ II: $\log \frac{x-1}{x+1} \geq 0 \rightarrow \frac{x-1}{x+1} \geq 1 \rightarrow \frac{x-1+x-1}{x+1} \geq 0 \rightarrow \frac{2x-2}{x+1} \geq 0 \rightarrow \frac{x-1}{x+1} \geq 0 \rightarrow x > 1$

$\frac{-1}{x+1} > 0 \rightarrow x < -1$ III: $\log \frac{x-1}{x+1} < 0 \rightarrow \frac{x-1}{x+1} < 1 \rightarrow \frac{x-1+x-1}{x+1} < 0 \rightarrow \frac{2x-2}{x+1} < 0 \rightarrow \frac{x-1}{x+1} < 0 \rightarrow x < 1$

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