

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$ $\rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0$
 $\rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{-2x + 1}{x(x-1)} \geq 0$
 $\rightarrow D_f = (-\infty, -\frac{1}{2}) \cup [1, +\infty)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}}$ $\rightarrow D_f = \{x \mid x \neq -1, x \neq -2\}$
 $\rightarrow D_f = \mathbb{R} \setminus \{-1, -2\}$

الف) $f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^x - 9\right)(32 - 4x)}$ $\rightarrow \left(\left(\frac{1}{x}\right)^x - 9\right)(32 - 4x) \geq 0$
 $\rightarrow \left(\frac{1}{x^x} - 9\right)(32 - 4x) \geq 0$
 $\rightarrow \frac{1}{x^x} - 9 \geq 0 \rightarrow \frac{1}{x^x} \geq 9 \rightarrow x^x \leq \frac{1}{9}$
 $\rightarrow x^x \leq \frac{1}{9} \rightarrow x \leq -1$
 $\rightarrow D_f = (-\infty, -1]$

$f(x) = \frac{\log_e(x^2 - x + 1)}{\sqrt{x^2 - 1} + 1}$ $\rightarrow x^2 - x + 1 > 0$
 $\rightarrow (x-2)(x+1) > 0$
 $\rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{4 + ax - x^2}$ $\rightarrow [-2, b]$
 $a + b = ?$
 $\rightarrow \frac{c}{a} = \frac{b}{a} \rightarrow \frac{4}{-1} = \frac{b}{-1} \rightarrow b = 4$
 $\rightarrow a + b = -2 + 4 = 2$

$f(x) = \begin{cases} 2x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases}$ $\rightarrow g(x) = \sqrt{f(x) - x}$
 $x \geq 1 \rightarrow f(x) - x = 2x - 2 - x = x - 2 \geq 0 \rightarrow x \geq 2$
 $x < 1 \rightarrow f(x) - x = 2x + 3 - x = x + 3 \geq 0 \rightarrow x \geq -3$
 $\rightarrow D_g = [-3, +\infty)$

$$f(x), x^2 - 4x$$

$$f\left(\sin \frac{\pi}{4} + \cos 2\pi\right) \rightarrow f\left(1 + \frac{\sqrt{2}}{2}\right) \xrightarrow{\text{محل}} f\left(1 + \frac{\sqrt{2}}{2}\right), 2\left(1 + \frac{\sqrt{2}}{2}\right)^2 - 4\left(1 + \frac{\sqrt{2}}{2}\right)$$

$$\left(\sin \frac{\pi}{4}, \frac{\sqrt{2}}{2} \quad \cos 2\pi, 1\right)$$

$$\sin \frac{\pi}{4} + \cos 2\pi = \frac{\sqrt{2}}{2} + 1$$

$$2\left(1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - 4 - 2\sqrt{2}$$

$$2 + \sqrt{2} + \sqrt{2} - 4 - 2\sqrt{2} = -2 - \sqrt{2}$$

$$f(x), \frac{x+1}{x-1} \rightarrow \text{دو تابع این تابع وارده}$$

$$f(f(a)) = a \rightarrow \text{فرض کنیم } (f(f(x)), x) \text{ حالت کلی}$$

$$a \in \mathbb{R} \setminus \{1\} \rightarrow a \neq 1$$

$$f(x), x^2 - 2x$$

$$g(x), \sqrt{x^2 - 1}, f(g(a)) = 2$$

$$a = ?$$

$$g(a)^2 - f(g(a)) = 2$$

$$(g(a) - 1)^2 = 0 \rightarrow g(a) = 1 \rightarrow \sqrt{a^2 - 1} = 1 \rightarrow a^2 - 1 = 1 \rightarrow a^2 = 2 \rightarrow a = \pm \sqrt{2}$$

$$f(x), x^2 - 8x + 9$$

$$f(f) \times f(-2) = ?$$

$$f(f), f(1+2), 1 - 8 + 9 = 2$$

$$f(-2), f(-9+2), 81 + 16 + 9 = 106$$

$$f(f) \times f(-2) = 2 \times 106 = 212$$

$$f(\cos^2 x + 2 \cos x), \tan^2 x$$

$$f(3) = 3 \checkmark$$

$$\Delta = 1.0 - 4(1)(-1) = 5 \rightarrow \sqrt{5}$$

$$\cos x = \frac{1 \pm \sqrt{5}}{2}$$

$$\cos x = \frac{1 + \sqrt{5}}{2} \rightarrow \cos x = \frac{1 + \sqrt{5}}{2}$$

$$\cos x = \frac{1 - \sqrt{5}}{2} \rightarrow \cos x = \frac{1 - \sqrt{5}}{2}$$

$$\cos x = \frac{1}{2} \rightarrow x = \frac{\pi}{3} \text{ یا } \frac{2\pi}{3}$$

$$\tan^2 x = 3 \rightarrow \tan x = \sqrt{3} \text{ یا } -\sqrt{3}$$

$$\tan \frac{\pi}{3} = \sqrt{3}$$

$$\tan \frac{2\pi}{3} = -\sqrt{3}$$