

حلینا محمود زاده با زدم ۵ - تکلیف ۱ - ۲. آفرین عالی زنده شکر

سوال ۱ - الف) $\frac{\sqrt{x-1}}{x} - \frac{x}{x-1} \geq \frac{x^2 - x + 1}{x^2 - x} \geq 0 \rightarrow \frac{-x+1}{x(x-1)} \geq 0$

Df = $(-\infty, 0) \cup [\frac{1}{2}, 1) \cup (1, +\infty)$

ب) $\frac{1}{x+1} - \frac{2}{x} \rightarrow x \neq 0 \rightarrow x \neq -1$

$\frac{x}{x-1} + \frac{1}{x+2} \rightarrow x \neq 1, x \neq -2$

Df = $\mathbb{R} - \left\{ \frac{1}{2}, -2, 1, -1, 0 \right\}$

$x+2+x-1 \neq 0 \rightarrow x \neq -\frac{1}{2}$

سوال ۲ - الف) $f(x) = \sqrt{\left(\left(\frac{1}{3}\right)^x - 9\right)(32 - 4^x)} \geq \frac{1}{3} - 9 \geq 0 \rightarrow \frac{1}{3} \geq 9 \rightarrow x \leq -2$

$32 - 4^x \geq 0 \rightarrow 32 \geq 4^x \rightarrow \log_4 32 \geq x \rightarrow 2.5$

Df = $(-\infty, -2] \cup [2.5, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} \geq 2$

$x-1 \geq 0 \rightarrow x \geq 1$

$\sqrt{y+1} \geq 2 - \sqrt{x-1} \rightarrow 2 - \sqrt{x-1} \geq 0 \rightarrow \sqrt{x-1} \leq 2 \rightarrow x-1 \leq 4 \rightarrow x \leq 5$

Df = $[1, 5]$

سوال ۳ - $\sqrt{x^2-1} \rightarrow x^2 \geq 1 \rightarrow x \geq 1 \vee x \leq -1$

Df = $(-\infty, -1) \cup (1, +\infty)$

$\sqrt{x^2-1} + 1 \neq 0 \rightarrow \sqrt{x^2-1} \neq -1$

$x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow x > 2 \vee x < -1$

Df = $(-\infty, -1) \cup (2, +\infty)$

سوال ۴ - $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0 \rightarrow \frac{1}{a} + \frac{1}{b} = -\frac{1}{c}$

$\frac{1}{a} \geq \frac{1}{b} \rightarrow \frac{1}{a} \geq \frac{1}{b} \rightarrow \frac{1}{a} \geq \frac{1}{b} \rightarrow \frac{1}{a} \geq \frac{1}{b} \rightarrow \frac{1}{a} \geq \frac{1}{b}$

$a+b = -\frac{1}{c} + \frac{1}{c} = 0$

$b = \frac{1}{c}$

سوال ۵ -

$$f'(x) - x \geq 0$$

$$x \geq 1 \text{ s.t. } 2x - x - x \geq 0 \Rightarrow x = 1 \rightarrow x \geq 1$$

$$x < 1 \text{ s.t. } 2x + 1 - x \geq 0 \rightarrow x \geq -1 \rightarrow D = [-1, +\infty)$$

سوال ۶ -

$$f(x) = 2x^2 - 4x$$

$$\sin \frac{\pi}{4} + \cos \frac{\pi}{4} = \left(t = \sqrt{\frac{5}{4}} + 1 \right)$$

$$f(t) = 2f\left(1 + \sqrt{\frac{5}{4}}\right)\left(\sqrt{\frac{5}{4}} - 1\right) = 2 \times \left(-\frac{1}{4}\right) = -\frac{1}{2}$$

$$f(a) = \frac{a+1}{a-1} \rightarrow f\left(\frac{a+1}{a-1}\right) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1+a-1}{a+1-a-1} = \frac{2a}{a-2} \rightarrow a \in \mathbb{R} - \{1\}$$

$$g(a) = \sqrt{a^2 - 1}$$

$$f(\sqrt{a^2 - 1}) = t^2 - 4t = 8 - 4 \rightarrow t^2 - 4t + 4 = 0$$

$$\frac{-b}{2a} = 2 \rightarrow \sqrt{a^2 - 1} = 2$$

$$a^2 - 1 = 4$$

سوال ۹ -

$$x + 2 = 1 \rightarrow x = -1 \rightarrow f(1) = 1 - 4 + 2 = -1$$

$$x + 2 = -1 \rightarrow x = -3 \rightarrow f(-1) = 1 - 4 + 2 = -1$$

$$f(x) = 1 - 4x + 2x^2$$

$$f\left(\frac{1}{2}\right) = \left(\cos^2 + \sin^2\right) \cos^2 \frac{1}{2}$$

$$\cos^2 + 1 - \cos^2 = 1 \rightarrow \cos^2 = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{2}$$

سوال ۱۰ -