

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$ $\rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{x^2-1-x^2}{x(x-1)} = \frac{-1}{x(x-1)} \geq 0$

$\frac{-1}{x(x-1)} \geq 0 \rightarrow D_f = (0, 1)$ $\frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{-2x+1}{x(x-1)} \geq 0$

$D_f = (-\infty, 0) \cup [\frac{1}{2}, +\infty)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{x}{x-1} + \frac{1}{x+1}}$

$x \neq -1, x \neq 0, x \neq 1, x \neq -2, \frac{x}{x-1} + \frac{1}{x+1} \neq 0 \rightarrow \frac{x^2+4x+1}{x^2+x-2} = \frac{x^2+4x+1}{x^2+x-2} \neq 0$

$x^2+4x+1 \neq 0 \rightarrow x \neq \frac{-4 \pm \sqrt{16-4}}{2} = \frac{-4 \pm \sqrt{12}}{2}$ $D_f = \mathbb{R} - \{-1, 0, 1, -2, \frac{-4 \pm \sqrt{12}}{2}\}$

الف) $f(x) = \sqrt{(\frac{1}{x})^2 - 9} = \sqrt{\frac{1}{x^2} - 9}$ $\rightarrow (\frac{1}{x})^2 - 9 \geq 0 \rightarrow \frac{1}{x^2} \geq 9 \rightarrow x^2 \leq \frac{1}{9} \rightarrow x \in [-\frac{1}{3}, \frac{1}{3}]$
 $D_f = (-\frac{1}{3}, \frac{1}{3}) \cup [\frac{1}{3}, +\infty)$
 $D_f = [-\frac{1}{3}, \frac{1}{3}]$
 $x=1$ مثبت است!

ب) $\sqrt{x-1} + \sqrt{y+1} \leq 3 \rightarrow \sqrt{y+1} \leq 3 - \sqrt{x-1} \rightarrow y+1 \leq 9 - 6\sqrt{x-1} + x-1 \rightarrow y \leq 8 - 6\sqrt{x-1} + x-1$
 $\sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow$ باید هر دو سمت مثبت باشند!
 $\sqrt{x-1} \leq 3 \rightarrow x-1 \leq 9 \rightarrow x \leq 10$
 $\sqrt{y+1} \geq 0 \rightarrow y+1 \geq 0 \rightarrow y \geq -1$
 $D_f = [1, 10]$
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$f(x) = \frac{\log(x^2-x-2)}{\sqrt{x^2-1} + 1} \rightarrow x^2-x-2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow x < -1 \text{ or } x > 2$
 $\sqrt{x^2-1} + 1 \neq 0 \rightarrow x^2-1 \geq 0 \rightarrow x \leq -1 \text{ or } x \geq 1$
 $D_f = (-\infty, -1] \cup [2, +\infty)$

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$f(x) = \sqrt{x^2+2x+3}$

$x^2+2x+3 \geq 0 \rightarrow x^2+2x+1 \geq -2 \rightarrow (x+1)^2 \geq -2$
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$(x^2+2x+3) = (x+1)^2 + 2 \geq 0$
 $(x^2+2x+3) = (x+1)^2 + 2 \geq 0$
 $(x^2+2x+3) = (x+1)^2 + 2 \geq 0$

$a = \frac{1}{x}$

$a+b = \frac{1}{x} + \frac{x}{x} = \frac{1+x^2}{x} \geq \frac{1}{x}$

$f(x) = \begin{cases} x^2-2 & x \geq 1 \\ x^2+2 & x < 1 \end{cases}$

$g(x) = \sqrt{f(x)} - x$
 $x \geq 1 \rightarrow \sqrt{x^2-2} - x \rightarrow \sqrt{x^2-2} \geq x \rightarrow x^2-2 \geq x^2 \rightarrow -2 \geq 0$
 $x < 1 \rightarrow \sqrt{x^2+2} - x \rightarrow \sqrt{x^2+2} \geq x \rightarrow x^2+2 \geq x^2 \rightarrow 2 \geq 0$
 $D_g = [-2, +\infty)$

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9 $f(x) = x^2 - 2x$

$f(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}) = f(\frac{\sqrt{2}}{2} + 1) = f(\frac{\sqrt{2}+2}{2}) \rightarrow \frac{(\frac{\sqrt{2}+2}{2})^2 - 2(\frac{\sqrt{2}+2}{2})}{2} = \frac{1-\sqrt{2}}{2}$

5 $f(f(a)) = a \rightarrow f(\frac{a+1}{a-1}) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1+a-1}{a+1-a+1} = \frac{2a}{2} = a$ ✓

$f(x) = \frac{x+1}{x-1}$
 $a-1 \neq 0 \rightarrow a \neq 1$
 $\frac{a+1}{a-1} - 1 \neq 0 \rightarrow \frac{a+1}{a-1} \neq 1 \rightarrow a-1 \neq a+1$ X $D_f = \mathbb{R} - \{1\}$ ✓

10 $f(x) = x^2 - 2x$ $x = \sqrt{a^2 - 1}$ $a^2 - 1 = (\sqrt{a^2 - 1})^2 - 2(\sqrt{a^2 - 1}) = 0 \rightarrow a^2 - 1 = 2\sqrt{a^2 - 1}$
 $g(x) = \sqrt{x^2 - 1}$ $a = \sqrt{a^2 - 1}$ $(\sqrt{a^2 - 1} - 2)^2 = 0$
 $f(g(a)) = -2$
 $\sqrt{a^2 - 1} = 2 \rightarrow a^2 - 1 = 4 \rightarrow a = \pm \sqrt{5}$ ✓

15 $f(x+y) = x^2 - 2x + y$
 $f(x) \times f(-x) = ? \rightarrow x^2 \times (-x^2) = -x^4$ ✓
 $f(-x) = x^2 - 2x + y = -x^2$
 $f(x) = 1 - 2 + y = x$

20 $f(\cos^2 \theta + \sin^2 \theta) = f(1) = 1$
 $f(\frac{1}{\cos^2 \theta}) = ? \rightarrow \frac{\sin^2 \theta}{\cos^2 \theta} \rightarrow \frac{\frac{1}{\cos^2 \theta} - 1}{\frac{1}{\cos^2 \theta} - 1} = 1$ ✓
 $f(\cos^2 \theta + \sin^2 \theta) = \frac{1}{\cos^2 \theta} = 1 \rightarrow \cos^2 \theta + \sin^2 \theta = 1 \rightarrow (\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta) = 1$
 $\cos^2 \theta = \frac{1}{2} \rightarrow \cos \theta = \pm \frac{1}{\sqrt{2}}$
 $\sin^2 \theta = \frac{1}{2} \rightarrow \sin \theta = \pm \frac{1}{\sqrt{2}}$

الف ۱) $\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{-2x+1}{x(x-1)} \geq 0$

$\begin{array}{c} \cdot \\ + \end{array} \begin{array}{c} \cdot \\ - \end{array} \begin{array}{c} \cdot \\ + \end{array} \begin{array}{c} \cdot \\ - \end{array} \begin{array}{c} \cdot \\ + \end{array} \begin{array}{c} \cdot \\ - \end{array}$
 $\rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب ۱) تمام ریشه‌های مخرج‌ها نباید در دامنه باشند!

$x+1 \neq 0 \rightarrow x \neq -1$ / $x \neq 0$ / $x-1 \neq 0 \rightarrow x \neq 1$ / $x+2 \neq 0 \rightarrow x \neq -2$ / $\frac{3}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{3x+4+x-1}{(x-1)(x+2)} \neq 0 \rightarrow 4x+3 \neq 0 \rightarrow x \neq -\frac{3}{4}$

$D_f = \mathbb{R} - \{-1, 0, 1, -2, -\frac{3}{4}\}$

الف ۲) $((\frac{1}{x})^n - 4)(3x - \varepsilon^n) \geq 0 \rightarrow \frac{-2}{+} \frac{0}{-} \rightarrow D_f = (-\infty, -2] \cup [\frac{a}{2}, +\infty)$

$(\frac{1}{x})^n = 4 \rightarrow x = -\frac{1}{2}$ $3x = \varepsilon^n \rightarrow x = \frac{\varepsilon}{3}$

$f(f(a)) = \frac{f(a)+1}{f(a)-1} = a \rightarrow \frac{\frac{a+1}{a-1}+1}{\frac{a+1}{a-1}-1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a$

$x \in D_f \rightarrow x \in \mathbb{R} - \{1\}$

پس به ازای تمامی نقاط در دامنه f بهر راست $\checkmark$

ب ۲) $\sqrt{x-1} + \sqrt{y+1} = 2$, $\sqrt{y+1} = 2 - \sqrt{x-1} \rightarrow 2 - \sqrt{x-1} \geq 0 \rightarrow \sqrt{x-1} \leq 2 \rightarrow x \leq 8$

$\downarrow$
 $x-1 \geq 0 \rightarrow x \geq 1$ I

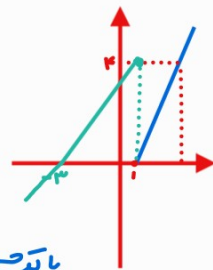
I, II $\rightarrow 1 \leq x \leq 8 \rightarrow D_f = [1, 8]$

$$f(n) - n = \begin{cases} 3n - 2 - n & n \geq 1 \\ 3n + 3 - n & n < 1 \end{cases} = \begin{cases} 2n - 2 & n \geq 1 \\ n + 3 & n < 1 \end{cases}$$

$$f(n) - n \geq 0 \rightarrow \begin{cases} 2n - 2 \geq 0 \rightarrow n \geq 1 \checkmark \\ n + 3 \geq 0 \xrightarrow{n < 1} -3 \leq n < 1 \checkmark \end{cases}$$

$$\rightarrow D_f = [-3, +\infty)$$

بابتوجه به سید نیز مشخص است!



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