

$$f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{x^2-1-x^2}{x(x-1)} \geq 0 \rightarrow \frac{-1}{x(x-1)} \geq 0$$

$$\frac{-1}{x(x-1)} \geq 0 \rightarrow D_f = (0, 1)$$

$$f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{x}{x-1} + \frac{1}{x+1}}$$

$$x \neq -1, x \neq 0, x \neq 1, x \neq -2, \frac{x}{x-1} + \frac{1}{x+1} \neq 0 \rightarrow \frac{x^2+4x+1}{x^2+x-1} \neq 0 \rightarrow \frac{x^2+4x+1}{x^2+x-1} \neq 0$$

$$x^2+4x+1 \neq 0 \rightarrow x \neq \frac{-4 \pm \sqrt{16-4}}{2} = \frac{-4 \pm \sqrt{12}}{2} = -2 \pm \sqrt{3}$$

$$D_f = \mathbb{R} - \{-1, 0, 1, -2 \pm \sqrt{3}\}$$

$$f(x) = \sqrt{\left(\frac{1}{x}\right)^2 - 9} \rightarrow \left(\frac{1}{x}\right)^2 - 9 \geq 0 \rightarrow \frac{1}{x^2} - 9 \geq 0 \rightarrow \frac{1-9x^2}{x^2} \geq 0 \rightarrow \frac{-1}{x^2} \geq 0 \rightarrow D_f = [-1, 1]$$

$$\sqrt{x-1} + \sqrt{y+1} \geq 3 \rightarrow \sqrt{y+1} \geq 3 - \sqrt{x-1} \rightarrow y+1 \geq 9 - 6\sqrt{x-1} + x-1 \rightarrow y \geq 8 - 6\sqrt{x-1} + x-1 \rightarrow y \geq x-5 + 6\sqrt{x-1}$$

$$x \geq 1 \rightarrow D_f = [1, +\infty)$$

$$f(x) = \frac{\log(x^2-x-2)}{\sqrt{x^2-1} + 1} \rightarrow x^2-x-2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow x < -1 \text{ or } x > 2$$

$$\sqrt{x^2-1} + 1 \neq 0 \rightarrow x^2-1 \geq 0 \rightarrow x \leq -1 \text{ or } x \geq 1 \rightarrow (-\infty, -1] \cup [1, +\infty)$$

$$f(x) = \sqrt{x^2+2x+2}$$

$$x^2+2x+2 \geq 0 \rightarrow x \geq -1 \rightarrow x \leq \frac{-2 \pm \sqrt{4-4}}{2} = -1 \rightarrow x \geq -1$$

$$(x^2+2x+2) = (x+1)^2 + 1 \geq 0 \rightarrow x \geq -1$$

$$a = \frac{1}{x} \rightarrow a+b = \frac{1}{x} + \frac{x}{x} = \frac{x+1}{x} \geq 1$$

$$f(x) = \begin{cases} x^2-2 & x \geq 1 \\ x^2+2 & x < 1 \end{cases}$$

$$g(x) = \sqrt{f(x)-x} \rightarrow \begin{cases} x^2-2-x \geq 0 & x \geq 1 \\ x^2+2-x \geq 0 & x < 1 \end{cases} \rightarrow \begin{cases} x^2-3x-2 \geq 0 & x \geq 1 \\ x^2-x+2 \geq 0 & x < 1 \end{cases}$$

$$D_g = \emptyset$$

Arman

Subject:

Year: Month: Day:

page: ()

$$f(x) = x^r - x$$

$$f\left(\sin \frac{\pi}{r} + \cos \frac{\pi}{r}\right) = f\left(\frac{\sqrt{r}}{r} + 1\right) = f\left(\frac{\sqrt{r}+r}{r}\right) \rightarrow \frac{Y(r+r+\sqrt{r})}{r^2} + \frac{-\sqrt{r}-1}{r} = \frac{r+r+\sqrt{r}}{r} - \frac{\sqrt{r}+1}{r} = \frac{r}{r} = 1$$

$$f(f(a)) = a \rightarrow f\left(\frac{a+1}{a-1}\right) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1+a-1}{a-1-a+1} = \frac{2a}{-2} = -a \checkmark$$

$$f(x) = \frac{x+1}{x-1}$$

$$a-1 \neq 0 \rightarrow a \neq 1$$

$$\frac{a+1}{a-1} - 1 \neq 0 \rightarrow \frac{a+1}{a-1} \neq 1 \rightarrow a-1 \neq a+1 \quad X \quad D_f = \mathbb{R} - \{1\}$$

$$f(x) = x^r - x \quad x = \sqrt{a^2-1} \quad a^2 - 1 - \sqrt{a^2-1} = f \rightarrow a^2 - \sqrt{a^2-1} = 0$$

$$g(x) = \sqrt{x^2-1} \quad a = x \quad \sqrt{a^2-1} \quad (\sqrt{a^2-1} - x)^2 = 0$$

$$f(g(a)) = -x$$

$$\sqrt{a^2-1} = x \rightarrow a^2 - 1 = x^2 \rightarrow a = \pm \sqrt{x^2+1}$$

$$f(x+r) = x^r - x + r$$

$$f(x) \times f(-x) = ? \rightarrow r \times \sqrt{r} = \sqrt{r^3}$$

$$f(-x) = x^r + x + r$$

$$f(x) = 1 - x + r$$

$$f(\cos^2 x + \sin^2 x) = f(1) = 1 - 1 = 0$$

$$f\left(\frac{r}{\cos^2 x}\right) = ? \rightarrow \frac{\sin^2 x}{\cos^2 x} \rightarrow \frac{r}{\cos^2 x} = \frac{r}{\cos^2 x}$$

$$f(\cos^2 x + \sin^2 x) = \frac{r}{\cos^2 x} = 0 \rightarrow \cos^2 x + \sin^2 x - \frac{r}{\cos^2 x} = 0 \rightarrow (\cos^2 x + \sin^2 x) \cos^2 x = r$$

$$\cos^2 x = \frac{r}{r+1} \quad \sin^2 x = \frac{1}{r+1}$$