

جوابی

$$f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \geq 0 \quad (1)$$

$$\frac{x-1}{x} - \frac{x}{x-1} = \frac{(x-1)^2 - x^2}{x(x-1)} = \frac{x^2 - 2x + 1 - x^2}{x(x-1)} = \frac{-2x + 1}{x(x-1)}$$

$$f(x) = \sqrt{\frac{1}{x+1} - \frac{1}{x}} \quad \begin{cases} x \neq 0 \\ x \neq -1 \end{cases} \quad R-1$$

$$\frac{1}{x-1} + \frac{1}{x+1} = \frac{x}{x-1} + \frac{1}{x+1} \neq 0 \rightarrow \frac{x^2 + 4 + x - 1}{(x-1)(x+1)} = \frac{x^2 + 3x + 3}{(x-1)(x+1)}$$

$$x^2 + 3x + 3 \neq 0 \quad \begin{cases} x \neq -1 \\ x \neq -3 \end{cases}$$

$$f(x) = \sqrt{\left(\frac{1}{x} - 1\right) \left(\frac{1}{x+1} - 1\right)} \geq 0$$

$$\left(\frac{1}{x} - 1\right) \left(\frac{1}{x+1} - 1\right) \geq 0 \rightarrow \frac{1-x}{x} \cdot \frac{1-x-1}{x+1} = 0$$

$$\frac{1}{x} - 1 = 0 \quad \begin{cases} x = 1 \\ x = -1 \end{cases} \quad \begin{cases} x \geq 1 \\ x \leq -1 \end{cases}$$

$$[x \geq -1]$$

$$y = \sqrt{x-1} + \sqrt{y+1} \quad (2)$$

$$y \geq 1 \quad \sqrt{y+1} \geq 1$$

$$x-1 \geq 0 \quad x \geq 1$$

$$\begin{cases} y \geq 0 \\ x \geq 1 \end{cases}$$

$$f(x) = \frac{1.0}{g} \left(\frac{x^2 - x - 1}{\sqrt{x^2 - 1} + 1} \right)$$

$x \geq 1$
 $x < -1$

$$\sqrt{x^2 - 1} + 1$$

$\frac{1}{\sqrt{x^2 - 1} + 1}$
 $\frac{1}{2}$

$$f(x) = \sqrt{r + ax - x^2} \geq 0 \quad [-b, b] \quad a + b \geq 0$$

$$\sqrt{-x^2 + ax + r} \geq 0 \Rightarrow -x^2 + ax + r \geq 0$$

$$-b^2 - \frac{1}{r}b + r \geq 0 \Rightarrow b^2 - \frac{1}{r}b + r \leq 0$$

$$f(x) = \sqrt{r + ax - x^2}$$

$x \geq 1$
 $x < 1$

$$f(x) - x \geq 0 \Rightarrow \sqrt{r + ax - x^2} - x \geq 0$$

$$f(x) \geq x$$

$$r + x \geq x \Rightarrow x \geq -r \Rightarrow -r \leq x < 1$$

$$f(x) = \sqrt{r + ax - x^2}$$

$$f\left(\sin \frac{\pi}{n}\right) = \sqrt{r + a \sin \frac{\pi}{n} - \sin^2 \frac{\pi}{n}}$$

$$\frac{r}{n} \left(\frac{r}{n} + 1 + \sqrt{\frac{r}{n}} \right) = \frac{r}{n} + r + r \sqrt{\frac{r}{n}} - r \sqrt{\frac{r}{n}} - \frac{r}{n}$$

مسائل

$$f(x) = \frac{x+1}{x-1} \rightarrow$$

$$f(f(a)) = a$$

$$\frac{a+1}{(a+1)(-a+1)} = \frac{a+1}{a-1} + 1 = a$$

$$\frac{(a+1)(a-1)}{(a+1)(-a+1)} \times \frac{(a-1)}{(a+1)(-a+1)} = a \rightarrow \frac{(a-1)}{(a+1)} = a$$

$$(a-1) = (-a+1)a \rightarrow a-1+a^2-a = 0 \Rightarrow a^2-1=0$$

$$a = \pm 1$$

$$f(x) = a - \frac{1}{a} \rightarrow a - \frac{1}{a} = x \Rightarrow a^2 - 1 = ax \Rightarrow a^2 - ax - 1 = 0$$

$$g(x) = \sqrt{ax^2 - 1}$$

$$f(g(a)) = \sqrt{a(a^2 - 1)} = \sqrt{a^3 - a} = \sqrt{a(a^2 - 1)} = \sqrt{a^3 - a}$$

$$f(a+x) = a+x - \frac{1}{a+x} \rightarrow a+x + \frac{1}{a+x} = a+1$$

$$f(f(x)) = f\left(a+x - \frac{1}{a+x}\right) = a+x + \frac{1}{a+x} = a+1$$

$$f(f(x)) = a+x + \frac{1}{a+x} = a+1$$

$$f(f(x)) = a+x + \frac{1}{a+x} = a+1 \Rightarrow \frac{1}{a+x} = a+1-x \Rightarrow 1 = (a+x)(a+1-x)$$