

(سوال 1)

$$الف) f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \geq 0 \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0$$

$$\rightarrow \frac{1-x}{x(x-1)} \leq 0 \rightarrow \frac{0}{-} + \frac{+}{-} = \frac{+}{+} \rightarrow Df = (-\infty, 0) \cup [\frac{1}{2}, 1)$$

$$ب) f(x) = \frac{1}{x+1} - \frac{2}{x} \geq 0 \rightarrow \frac{1}{x+1} \geq \frac{2}{x} \rightarrow x+1 \neq 0 \rightarrow x \neq -1, x \neq 0, \frac{1}{x-1} + \frac{1}{x+1} \neq 0$$

$$\rightarrow \frac{1(x+1) + (x-1)}{(x+1)(x-1)} \neq 0 \rightarrow \frac{2x}{(x+1)(x-1)} \neq 0 \rightarrow x \neq -\frac{1}{2}, -1, 1$$

$$Df = \mathbb{R} - \left\{ -\frac{1}{2}, -1, 1 \right\}$$

(سوال 2)

$$الف) f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^2 - 9\right)(32 - 4^x)} \geq 0 \rightarrow \left(\left(\frac{1}{x}\right)^2 - 9\right)(32 - 4^x) \geq 0$$

$$\left(\frac{1}{x^2} - 9\right)(32 - 2^{2x}) \geq 0 \rightarrow \frac{1}{x^2} - 9 = 0 \rightarrow x = -\frac{1}{3}$$

$$32 - 2^{2x} = 0 \rightarrow x = \frac{\log 32}{\log 4} \rightarrow Df = (-\infty, -\frac{1}{3}] \cup [\frac{\log 32}{\log 4}, +\infty)$$

$$ب) \sqrt{x-1} + \sqrt{y+1} = 2 \rightarrow x-1 \geq 0 \rightarrow x \geq 1, y+1 \geq 0 \rightarrow y \geq -1$$

$$\sqrt{y+1} = 2 - \sqrt{x-1} \rightarrow 2 - \sqrt{x-1} \geq 0 \rightarrow \sqrt{x-1} \leq 2 \rightarrow x-1 \leq 4 \rightarrow x \leq 5$$

$$\textcircled{1} \cap \textcircled{2} \rightarrow Df = [1, 5]$$

$$\textcircled{1} (x \leq 5)$$

$$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$$

$$\rightarrow x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \quad \text{(سؤال ١٣)}$$

$$\rightarrow x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq 1 \text{ و } x \leq -1 \quad \text{(١)}$$

$$\rightarrow \sqrt{x^2 - 1} + 1 \neq 0 \sim \mathbb{R} \quad \text{(٢)}$$

$$\textcircled{1} \cap \textcircled{2} \cap \textcircled{3} \rightarrow Df = (-\infty, -1) \cup (2, +\infty)$$

$$f(x) = \sqrt{3 + ax - x^2} \rightarrow 3 + ax - x^2 \geq 0 \rightarrow x^2 - ax - 3 \leq 0 \quad \text{(سؤال ١٤)}$$

$$Df = [r, b] \rightarrow 3 + ra - r^2 = 0 \rightarrow a = -\frac{3}{r}$$

$$\rightarrow a = -\frac{3}{r} + b \rightarrow -\frac{3}{r} + r = b \rightarrow b = \frac{r^2 - 3}{r}$$

$$a + b = -\frac{3}{r} + \frac{r^2 - 3}{r} = \frac{r^2 - 6}{r} = \frac{r^2}{r} = r$$

$$f(x) = \begin{cases} 3x - 2 & x \geq 1 \\ 2x + 3 & x < 1 \end{cases} \quad \text{(سؤال ١٥)}$$

$$g(x) = \sqrt{f(x) - x} \rightarrow f(x) - x \geq 0 \rightarrow f(x) \geq x$$

$$3x - 2 \geq x \rightarrow 2x \geq 2 \rightarrow x \geq 1 \rightarrow [1, +\infty) \quad \text{(١)}$$

$$2x + 3 \geq x \rightarrow x \geq -3 \rightarrow (-\infty, 1) \cap [-3, +\infty) \rightarrow [-3, 1) \quad \text{(٢)}$$

$$\rightarrow \textcircled{1} \cup \textcircled{2} \rightarrow Dg = [-3, +\infty)$$

$$f(x+y) = x^2 - \omega x + y$$

(9 سوال)

$$f(x) \xrightarrow{n=1} f(1+y) = 1^2 - (\omega \times 1) + y = 1 - \omega + y = \boxed{2}$$

$$f(-y) \xrightarrow{n=-4} f(\underbrace{-4+y}_{-y}) = (-4)^2 - (\omega \times -4) + y = 16 + 4\omega + y = \boxed{17}$$

$$f(x) \times f(-y) = 2 \times 17 = \boxed{34}$$

$$f(1 + \cos^2 m + \omega \cos m) = \tan^2 m$$

(10 سوال)

$$f(1/\omega) = ?$$

$$1 + \cos^2 m + \omega \cos m = 1/\omega \rightarrow 1 + \cos^2 m + \omega \cos m - \frac{1}{\omega} = 0$$

$$\rightarrow \Lambda \cos^2 m + 1 \cos m - \frac{1}{\omega} = 0 \xrightarrow{(\text{G})} \cos^2 m + 1 \cos m - \omega = 0$$

$$\rightarrow (\cos m + 1)(\cos m - \omega) = 0$$

$$\rightarrow \cos m = +\frac{1}{\omega} \text{ , } \underbrace{-\frac{1}{\omega}}_{\substack{-1 < \cos m < 1 \\ 0 < \omega < 1}} \rightarrow \cos m = \frac{1}{\omega} \rightarrow \tan^2 m = \left(\frac{\sin m}{\cos m} \right)^2$$

$\downarrow$
 $m = 90^\circ$

$$\tan^2 m = \left(\frac{\frac{\sqrt{3}}{\omega}}{\frac{1}{\omega}} \right)^2 = (\sqrt{3})^2 = \boxed{3}$$

