

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \sqrt{\frac{(x-1)^2 - x^2}{(x-1)x}} = \sqrt{\frac{(x-1-x)(x-1+x)}{(x-1)x}} = \sqrt{\frac{1-2x}{x(x-1)}} \Rightarrow$

$\frac{0}{+} - \frac{1}{-} + \frac{1}{+} -$ $D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+2}}$ $I: x \neq -1, x \neq 0$ $II: \frac{1}{x-1} + \frac{1}{x+2} \neq 0 \Rightarrow$

$\frac{1(x+2) - (x+1)}{(x-1)(x+2)} \neq 0 \Rightarrow \frac{x+2-x-1}{(x-1)(x+2)} \neq 0 \Rightarrow \frac{1}{(x-1)(x+2)} \neq 0$ $D_f = \mathbb{R} - \left\{ -2, -\frac{1}{2}, -1, 0, 1 \right\}$

الف) $f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^x - 9\right)(x^2 - x^x)}$ $\left(\left(\frac{1}{x}\right)^x - 9\right)(x^2 - x^x) \geq 0$ $\left(\frac{1}{x}\right)^x \geq 9 \Rightarrow x \leq -2$

$\frac{-2}{+} - \frac{1}{-} + \frac{1}{+} -$ $D_f = (-\infty, -2] \cup [2, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3$ $I: x-1 \geq 0 \rightarrow x \geq 1$ $II: \sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow 3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$

$II \cap I \Rightarrow 1 \leq x \leq 10$ $D_f = [1, 10]$ $\sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$

$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$ $I: x^2 - x - 2 > 0 \rightarrow (x+1)(x-2) > 0$ $\frac{-1}{+} - \frac{1}{-} + \frac{1}{+} - \rightarrow (-\infty, -1) \cup (2, +\infty)$

$II: x^2 - 1 > 0 \rightarrow x^2 > 1 \rightarrow x > 1, x < -1 \rightarrow (-\infty, -1] \cup [1, +\infty)$

$III: \sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \sqrt{x^2 - 1} \neq -1 \rightarrow \mathbb{R}$ $II \cap I: \frac{(-\infty, -1) \cup (2, +\infty)}{(-\infty, -1] \cup [1, +\infty)} \Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{x^2 + ax - x^2}$ $D_f = [-2, b]$ $\frac{x^2 + ax - x^2}{-1 - ax} \geq 0 \rightarrow xa = -1 \rightarrow a = -\frac{1}{x}$

$f(x) = \sqrt{x^2 - \frac{1}{x} - x^2}$ $x^2 - \frac{1}{x} - x^2 \geq 0 \rightarrow x^2 + \frac{1}{x} - x^2 \leq 0$ $x^2 > 2x^2 + x - 9 \leq 0 \rightarrow x^2 - 2x - 12 \leq 0$

$(x-4)(x+3) \leq 0 \rightarrow x = -3$ $(x-4)(x+3) \leq 0 \rightarrow x = 4$ $D_f = [-2, \frac{1}{2}] \rightarrow b = \frac{1}{2}$ $a+b = \frac{1}{x} - \frac{1}{x} = 0$

$f(x) = \begin{cases} 2x-2 & x \geq 1 \\ 2x+2 & x < 1 \end{cases}$ $g(x) = \sqrt{f(x)-x}$ $\textcircled{1} x \geq 1: f(x)-x = 2x-2-x = x-2 \geq 0 \rightarrow x \geq 2 \rightarrow [2, +\infty)$

$\textcircled{2} x < 1: f(x)-x = 2x+2-x = x+2 \geq 0, x+2 \geq 0 \rightarrow x \geq -2 \rightarrow [-2, 1)$ $D_f = [-2, +\infty)$

$f(x) = 2x^2 - 2x$ $f\left(\frac{\sin \frac{\pi}{4}}{\frac{\sqrt{3}}{2}} + \cos \frac{\pi}{4}\right) \Rightarrow f\left(\frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} + 1\right) = 2\left(\frac{\sqrt{3}}{2} + 1\right)\left(\frac{\sqrt{3}}{2} + 1 - 2\right) =$

$(\sqrt{3}+2)\left(\frac{\sqrt{3}}{2} - 1\right) = \frac{1}{2}(\sqrt{3}+2)(\sqrt{3}-2) = \frac{1}{2}(3-4) = -\frac{1}{2}$

$$f(x) = \frac{x+1}{x-1}$$

$$f(f(a)) = a$$

$$a=1 \rightarrow f(1) \rightarrow \infty \rightarrow f(\infty) \rightarrow 1$$

$$a=0 \rightarrow f(0) \rightarrow -1 \rightarrow f(-1) \rightarrow 0$$

$$a=-1 \rightarrow f(-1) = \frac{1}{-1} \rightarrow f\left(\frac{1}{-1}\right) = -1$$

$$f(f(a)) = \frac{\frac{a+1}{a-1}+1}{\frac{a+1}{a-1}-1} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a$$

$$a \in \mathbb{R} - \{1\}$$

برای مقادیر a به جز 1 صبیح است چون 1 مخرج را صفر و تعریف نشده می کند

$$f(x) = x^2 - 4x$$

$$f(g(a)) = -4$$

$$x^2 - 4x = -4 \rightarrow x^2 - 4x + 4 = 0 \Rightarrow$$

$$(x-2)^2 = 0 \rightarrow x = 2 \Rightarrow g(a) = 2 \Rightarrow$$

$$\sqrt{a^2 - 1} = 2 \Rightarrow a^2 - 1 = 4 \Rightarrow a^2 = 5 \Rightarrow a = \pm \sqrt{5}$$

$$g(x) = \sqrt{x^2 - 1}$$

$$a = \pm \sqrt{5}$$

$$f(x+1) = x^2 - 4x + 4$$

$$f(1) = 1 - 4 + 4 = 1$$

$$x+1 = 1 \rightarrow x = 0$$

$$x+1 = -1 \rightarrow x = -2$$

$$f(-1) = 1 - 4 + 4 = 1$$

$$f(1) = f(-1) = 1$$

$$f(\sqrt{1+\cos^2 x} + \sin x) = \tan^2 x$$

$$f(1/\omega) = 1$$

$$\sqrt{1+\cos^2 x} + \sin x = 1/\omega \rightarrow \sqrt{1+\cos^2 x} + \sin x - \frac{1}{\omega} = 0 \rightarrow \sqrt{1+\cos^2 x} + \sin x - 1 = 0 \Rightarrow$$

$$(\cos x - 1)(\cos x + 1) = 0 \Rightarrow \begin{cases} \cos x = 1 \checkmark \rightarrow x = 0 \Rightarrow \tan^2 0 = 0 \\ \cos x = -1 \checkmark \rightarrow x = \pi \Rightarrow \tan^2 \pi = 0 \end{cases}$$

$$f(1/\omega) = 1$$