

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \sqrt{\frac{(x-1)^2 - x^2}{(x-1)x}} = \sqrt{\frac{(x-1-x)(x-1+x)}{(x-1)x}} = \sqrt{\frac{1-2x}{x(x-1)}} \Rightarrow$
 $\frac{0}{+} - \frac{1}{-} + \frac{1}{+} -$ $D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+2}}$ $I: x \neq -1, x \neq 0$ $II: \frac{1}{x-1} + \frac{1}{x+2} \neq 0 \Rightarrow$

$\frac{1(x+2) + (x-1)}{(x-1)(x+2)} \neq 0 \Rightarrow \frac{2x+1}{(x-1)(x+2)} \neq 0 \Rightarrow \frac{2x+1}{(x-1)(x+2)} \neq 0$ $D_f = \mathbb{R} - \left\{ -2, -\frac{1}{2}, -1, 0, 1 \right\}$

الف) $f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^x - 9\right)(x^2 - x^x)}$ $\left(\left(\frac{1}{x}\right)^x - 9\right)(x^2 - x^x) \geq 0$ $\left(\frac{1}{x}\right)^x \geq 9 \Rightarrow x \leq -2$
 $\frac{-2}{+} - \frac{1}{-} + \frac{1}{+} -$ $D_f = (-\infty, -2] \cup [2, +\infty)$ $x^2 = x^2 \rightarrow x^{2x} = x^2 \rightarrow x^2 \geq 0 \Rightarrow x \geq \frac{1}{2}$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3$ $I: x-1 \geq 0 \rightarrow x \geq 1$ $II: \sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow 3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$
 $II \cap I \Rightarrow 1 \leq x \leq 10$ $D_f = [1, 10]$ $\sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$

$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$ $I: x^2 - x - 2 \geq 0 \rightarrow (x+1)(x-2) \geq 0$ $\frac{-1}{+} - \frac{2}{-} + \frac{1}{+} - \rightarrow (-\infty, -1) \cup (2, +\infty)$
 $II: x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq 1, x \leq -1 \rightarrow (-\infty, -1] \cup [1, +\infty)$ $III: \sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \sqrt{x^2 - 1} \neq -1 \rightarrow \mathbb{R}$ $II \cap I: \frac{-1}{+} - \frac{2}{-} + \frac{1}{+} - \rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$f(x) = \sqrt{x^2 + ax - x^2}$ $D_f = [-2, b]$ $\frac{x^2 + ax - x^2}{-1 - ax} \geq 0 \rightarrow xa = -1 \rightarrow a = -\frac{1}{x}$
 $f(x) = \sqrt{x^2 - \frac{1}{x}x - x^2}$ $x^2 - \frac{1}{x}x - x^2 \geq 0 \rightarrow x^2 + \frac{1}{x}x - x^2 \leq 0$ $x^2 \geq 2x^2 + x - 9 \leq 0 \rightarrow x^2 + x - 12 \leq 0$
 $(x-3)(x+4) \leq 0 \rightarrow x = -4$ $\frac{-4}{+} - \frac{1}{-} + \frac{1}{+} - \rightarrow D_f = [-2, \frac{1}{2}] \rightarrow b = \frac{1}{2}$ $a+b = \frac{1}{x} - \frac{1}{x} = 0$

$f(x) = \begin{cases} x^2 - 2 & x \geq 1 \\ x^2 + 2 & x < 1 \end{cases}$ $g(x) = \sqrt{f(x) - x}$ ① $x \geq 1: f(x) - x = x^2 - 2 - x = x^2 - x - 2$
 $x^2 - x - 2 \geq 0 \rightarrow x \geq 1 \rightarrow [1, +\infty)$ ② $x < 1: f(x) - x = x^2 + 2 - x \geq x + 2, x + 2 \geq 0 \rightarrow x \geq -2 \rightarrow [-2, 1)$ $D_g = [-2, +\infty)$

$f(x) = x^2 - 2x$ $f(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}) \Rightarrow f(\frac{\sqrt{2}}{2} + 1) = 2(\frac{\sqrt{2}}{2} + 1) \left(\frac{\sqrt{2}}{2} + 1 - 2 \right) =$
 $(\sqrt{2} + 2) \left(\frac{\sqrt{2}}{2} - 1 \right) = \frac{1}{2} \sqrt{2} + \sqrt{2} - 2 = -\frac{1}{2}$

$$f(n) = \frac{n+1}{n-1}$$

$$f(f(a)) = a$$

$$a=1 \rightarrow f(1) \rightarrow \infty \rightarrow f(\infty) \rightarrow 1$$

$$a=0 \rightarrow f(0) \rightarrow -1 \rightarrow f(-1) \rightarrow 0$$

$$a=-1 \rightarrow f(-1) = \frac{1}{-2} \rightarrow f\left(\frac{1}{-2}\right) = -1$$

$$f(f(a)) = \frac{\frac{a+1}{a-1}+1}{\frac{a+1}{a-1}-1} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a \quad a \in \mathbb{R} - \{1\}$$

برای مقادیر از a به جز 1 صحیح است چون 1 مخرج را صفر و تعریف نشده می کند

$$f(n) = n^2 - 4n$$

$$f(g(a)) = -4$$

$$n^2 - 4n = -4 \rightarrow n^2 - 4n + 4 = 0 \Rightarrow$$

$$(n-2)^2 = 0 \rightarrow n = 2 \Rightarrow g(a) = 2 \Rightarrow$$

$$\sqrt{a^2 - 1} = 2 \Rightarrow a^2 - 1 = 4 \Rightarrow a^2 = 5 \Rightarrow a = \pm \sqrt{5}$$

$$g(n) = \sqrt{n^2 - 1}$$

$$a = \pm \sqrt{5}$$

$$f(x+1) = x^2 - 4x + 4$$

$$f(1) = 1 - 4 + 4 = 1$$

$$x+1=1 \rightarrow x=0$$

$$x+1=-1 \rightarrow x=-2$$

$$f(-1) = 1 - 4 + 4 = 1$$

$$f(1) = f(-1) = 1$$

$$f(\sqrt{1+\cos^2 x} + \sin x) = \tan^2 x$$

$$f(1/\omega) = 1$$

$$\sqrt{1+\cos^2 x} + \sin x = 1/\omega \rightarrow \sqrt{1+\cos^2 x} + \sin x - \frac{1}{\omega} = 0 \rightarrow \cos^2 x + \sin^2 x - 1/\omega^2 = 0 \Rightarrow$$

$$(\cos x - 1)(\cos x + 1) = 0 \Rightarrow \begin{cases} \cos x = \frac{1}{\omega} \checkmark \rightarrow x = \arccos\left(\frac{1}{\omega}\right) \Rightarrow \tan^2 x = \omega^2 - 1 \\ \cos x = -\frac{1}{\omega} = -1/\omega \text{ not possible} \end{cases}$$

$$f(1/\omega) = \omega^2 - 1$$