

۲. افشین علی نوری

الف:

$$f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$$

$$\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{x^2-1-x^2}{x(x-1)} \geq 0$$

$$\frac{-1}{x(x-1)} \geq 0 \rightarrow \frac{0}{x(x-1)} \geq 0$$

$$D_f = (-\infty, 0) \cup \left[\frac{1}{x}, 1\right)$$

ب:

$$f(x) = \frac{1}{x+1} - \frac{1}{x-1}$$

$$D_f = \mathbb{R} - \left[-1, -\frac{1}{2}, 0, 1, -\frac{1}{x}\right]$$

ج:

$$\frac{1}{x+1} - \frac{1}{x-1} = \frac{x-1-x-1}{(x+1)(x-1)} = \frac{-2}{(x+1)(x-1)}$$

$$x \neq -1, 0, 1, -\frac{1}{2}, -\frac{1}{x}$$

الف:

$$f(x) = \sqrt{\left[\left(\frac{1}{x}\right)^x - 9\right] \left[3x - x^2\right]}$$

$$\left[\left(\frac{1}{x}\right)^x - 9\right] \left[3x - x^2\right] \geq 0$$

$$\left(\frac{1}{x}\right)^x - 9 \geq 0$$

$$3x - x^2 \geq 0$$

$$D_f = (-\infty, -\frac{1}{2}) \cup \left[\frac{1}{2}, +\infty\right)$$

ب:

$$\sqrt{x+1} + \sqrt{y+1} = 3$$

$$D_f = [1, 12]$$

ج:

$$a = \sqrt{x+1} \rightarrow a+b=3 \rightarrow a=3-b$$

$$b = \sqrt{y+1}$$

$$0 \leq a \leq 3$$

$$0 \leq b \leq 3$$

$$0 \leq \sqrt{x+1} \leq 3$$

$$0 \leq \sqrt{y+1} \leq 3$$

$$0 \leq x-1 \leq 11 \rightarrow 1 \leq x \leq 12$$

الف:

$$\frac{\log(x^2-x-2)}{\sqrt{x^2-1}+1}$$

$$D_f = (-\infty, -1) \cup (1, +\infty)$$

ب:

$$\frac{x^2-x-2}{(x-2)(x+1)} > 0$$

$$\frac{-1}{x-1} > 0 \Rightarrow (-\infty, -1) \cup (1, +\infty)$$

ج:

$$\sqrt{x^2-1} \Rightarrow x^2-1 \geq 0$$

$$x^2 \geq 1$$

$$x \geq 1 \text{ یا } x \leq -1$$

$$[-1, +\infty) \cup (-\infty, -1]$$

د:

$$\sqrt{x^2-1} + 1 \geq 0$$

$$\sqrt{x^2-1} \geq -1$$

$$R$$

الف:

$$f(x) = \sqrt{3+ax-x^2} \Rightarrow 3+ax-x^2 \geq 0$$

$$D_f = [-\frac{1}{2}, b]$$

ب:

$$a+b=?$$

$$-\frac{1}{2} + \frac{3}{2} = 1$$

ج:

$$x=-\frac{1}{2} \rightarrow 3 - \frac{1}{4} - \frac{1}{4} \geq 0$$

$$- \frac{1}{2} a \geq 1$$

$$a \leq -\frac{1}{2}$$

$$a = -\frac{1}{2}$$

د:

$$p = \frac{3}{a} = -2 \times b$$

$$b = \frac{3}{2}$$

الف:

$$f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 4x+2 & x < 1 \end{cases}$$

ب:

$$f(x) = \sqrt{f(x)-x}$$

$$f(x)-x \geq 0 \rightarrow f(x) \geq x$$

ج:

$$x \geq 1$$

$$3x-2-x \geq 0$$

$$2x-2 \geq 0$$

$$x \geq 1$$

د:

$$x < 1$$

$$4x+2-x \geq 0$$

$$3x+2 \geq 0$$

$$x \geq -\frac{2}{3}$$

$$D_f = \left[-\frac{2}{3}, +\infty\right)$$

$$f(x) = x^2 - \sqrt{x} \Rightarrow x(\frac{\sqrt{x}}{x} + 1)^2 - f(\frac{\sqrt{x}}{x} + 1) = x(\frac{x}{x^2} + 1 + \sqrt{x}) - f(\frac{\sqrt{x}}{x} + 1) =$$

$$f(\sin \frac{\pi}{4} + \cos \pi)$$

$$\frac{x}{x} + x + \sqrt{x} - \sqrt{x} - \sqrt{x} = -\frac{1}{x}$$

$$\sin \frac{\pi}{4} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos \pi = -1$$

$$\Rightarrow x = \frac{\sqrt{2}}{2} + 1$$

$$f(x) = \frac{x+1}{x-1} \Rightarrow f(a) = \frac{a+1}{a-1} \Rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a$$

$$f(f(a)) = a$$

$$\frac{a+1}{a-1} + 1 = \frac{a+1+a-1}{a-1} = \frac{2a}{a-1}$$

$$\frac{a+1}{a-1} - 1 = \frac{a+1-a+1}{a-1} = \frac{2}{a-1}$$

$$\frac{2a}{\frac{2}{a-1}} = a$$

معما تقریباً نه است
به قدر جابجایی تا به تقریب نه شود
Df = R - [1]

$$f(x) = x^2 - \sqrt{x} \Rightarrow f(g(a)) = (\sqrt{a-1})^2 - \sqrt{\sqrt{a-1}} + \sqrt{x} = 0 \Rightarrow \sqrt{a-1} = t$$

$$g(x) = \sqrt{x^2 - 1} \Rightarrow g(a) = \sqrt{a^2 - 1}$$

$$t^2 - \sqrt{t} + \sqrt{x} = 0 \Rightarrow (t-2)^2 = 0$$

$$t = 2$$

$$f(g(a)) = -\sqrt{x}$$

$$\sqrt{a^2 - 1} = 2$$

$$a^2 - 1 = 4$$

$$a^2 = 5$$

$$a = \pm \sqrt{5}$$

$$f(x+y) = x^2 - 2x + y$$

$$f(x) \propto f(-x) \Rightarrow x+y = -x \Rightarrow x = -y$$

$$x+y = 1 \Rightarrow x = 1$$

$$y^2 + y + y = y^2$$

$$f(1) = 1 - 1 + y = y$$

$$f(\cos x + a \cos x) = \tan x$$

$$f(\frac{1}{2}) = ? \rightarrow \cos x = \frac{1}{2} \rightarrow \tan x = \sqrt{3} \rightarrow f(\frac{1}{2}) = \sqrt{3}$$

↓

$$\cos x + a \cos x = \frac{1}{2} \Rightarrow \cos x = t \Rightarrow t^2 + at - \frac{1}{2} = 0 \Rightarrow 1t^2 + 10t - 1 = 0$$

$$t = \frac{-10 \pm \sqrt{100 + 4}}{2} = \frac{-10 \pm 10.2}{2}$$

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Df = R - [1]