

الف:

$$f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$$

$$\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{x^2+1-x^2}{x(x-1)} \geq 0$$

$$\frac{1-x^2}{x(x-1)} \geq 0 \rightarrow \frac{0}{\frac{1}{x} - 1 + \frac{1}{x}} \geq 0$$

ب:

$$f(x) = \frac{1}{x+1} - \frac{1}{x-1}$$

$$D_f = \mathbb{R} - [-1, -2, 0, 1, -\frac{1}{2}]$$

ج:

$$\frac{1}{x+1} - \frac{1}{x-1} = \frac{x-1-x-1}{x(x-1)} = \frac{-x-2}{x(x-1)}$$

$$D_f = (-\infty, 0) \cup (\frac{1}{2}, 1)$$

الف:

$$f(x) = \sqrt{[(\frac{1}{x})^x - 9][x^2 - x^x]}$$

$$[(\frac{1}{x})^x - 9][x^2 - x^x] \geq 0$$

$$(\frac{1}{x})^x - 9 \geq 0 \rightarrow x^x \geq 9$$

$$x^x \geq 9 \rightarrow x \leq -2$$

$$D_f = (-\infty, -2] \cup [\frac{9}{2}, +\infty)$$

ب:

$$\sqrt{x+1} + \sqrt{y+1} = 3$$

$$D_f = [1, 12]$$

ج:

$$a = \sqrt{x+1}, b = \sqrt{y+1}$$

$$a+b=3 \rightarrow a=3-b$$

$$0 \leq a \leq 3, 0 \leq b \leq 3$$

$$0 \leq \sqrt{x+1} \leq 3 \rightarrow -1 \leq x \leq 8$$

$$0 \leq \sqrt{y+1} \leq 3 \rightarrow -1 \leq y \leq 8$$

الف:

$$\frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$$

$$D_f = (-\infty, -1) \cup (2, +\infty)$$

ب:

$$\log(x^2 - x - 2) \geq 0$$

$$(x-2)(x+1) \geq 0 \Rightarrow (-\infty, -1) \cup (2, +\infty)$$

ج:

$$\sqrt{x^2 - 1} + 1 \geq 0$$

$$\sqrt{x^2 - 1} \geq -1$$

$$D_f = \mathbb{R}$$

الف:

$$f(x) = \sqrt{x+ax-x^2} \Rightarrow x+ax-x^2 \geq 0$$

$$D_f = [-2, b]$$

$$a+b=?$$

$$-\frac{1}{2} + \frac{3}{2} = 1$$

ب:

$$x-y \rightarrow x-2a-x \geq 0$$

$$-2a \geq 1$$

$$a \leq -\frac{1}{2}$$

$$a = -\frac{1}{2}$$

ج:

$$p = \frac{x}{a} = -2 \times b$$

$$b = \frac{3}{2}$$

الف:

$$f(x) = \begin{cases} x^2 - 2 & x \geq 1 \\ x^2 + 2 & x < 1 \end{cases}$$

$$g(x) = \sqrt{f(x) - x}$$

$$f(x) - x \geq 0 \rightarrow f(x) \geq x$$

ب:

$$x \geq 1$$

$$x^2 - 2 - x \geq 0$$

$$x^2 - x - 2 \geq 0$$

$$x \geq 2$$

ج:

$$x < 1$$

$$x^2 + 2 - x \geq 0$$

$$x^2 - x + 2 \geq 0$$

$$x \geq \frac{1}{2}$$

$$D_f = [-\frac{1}{2}, +\infty)$$

$$f(x) = x^2 - \sqrt{x} \Rightarrow x(\frac{\sqrt{x}}{x} + 1)^2 - \sqrt{\frac{\sqrt{x}}{x} + 1} = x(\frac{x}{x^2} + 1 + \sqrt{x}) - \sqrt{\frac{\sqrt{x}}{x} + 1} =$$

$$f(\sin \frac{\pi}{4} + \cos \pi) \quad \frac{x}{x} + x + \sqrt{x} - \sqrt{x} - \sqrt{x} = -\frac{1}{x}$$

$$\sin \frac{\pi}{4} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos \pi = -1$$

$$\Rightarrow x = \frac{\sqrt{2}}{2} + 1$$

$$f(x) = \frac{x+1}{x-1} \Rightarrow f(a) = \frac{a+1}{a-1} \Rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = a$$

$$f(f(a)) = a$$

$$\frac{a+1}{a-1} + 1 = \frac{a+1+a-1}{a-1} = \frac{2a}{a-1}$$

$$\frac{a+1}{a-1} - 1 = \frac{a+1-a+1}{a-1} = \frac{2}{a-1}$$

$$\frac{2a}{\frac{2}{a-1}} = a$$

اگر دو عبارت را در یک جا قرار دهیم و آن را ساده کنیم به دست می آوریم a

$$f(x) = x^2 - \sqrt{x} \Rightarrow f(g(a)) = (\sqrt{a-1})^2 - \sqrt{\sqrt{a-1}} + \sqrt{x} = 0 \Rightarrow \sqrt{a-1} = t$$

$$g(x) = \sqrt{x^2 - 1} \Rightarrow g(a) = \sqrt{a^2 - 1}$$

$$t^2 - \sqrt{t} + \sqrt{x} = 0 \Rightarrow (t-2)^2 = 0$$

$$t = 2$$

$$f(g(a)) = -\sqrt{x}$$

$$\sqrt{a^2 - 1} = 2$$

$$a^2 - 1 = 4$$

$$a^2 = 5$$

$$a = \pm \sqrt{5}$$

$$f(x+y) = x^2 - 2x + y$$

$$f(x) = f(-x) \Rightarrow x+y = -x \Rightarrow x = -y$$

$$x+y = 1 \Rightarrow x = 1$$

$$y^2 + y + y = y^2$$

$$f(x) = 1 - 1 + y = y$$

$$f(\cos x + \omega \cos x) = \tan x$$

$$f(\frac{1}{\omega}) = ? \Rightarrow \cos x = \frac{1}{\omega} \Rightarrow \tan x = \omega \Rightarrow f(\frac{1}{\omega}) = \omega$$

↓

$$\cos x + \omega \cos x = \frac{1}{\omega} \Rightarrow \cos x = t \Rightarrow t^2 + \omega t - \frac{1}{\omega} = 0 \Rightarrow 1t^2 + \omega t - \frac{1}{\omega} = 0$$

$$-\frac{1}{\omega} \pm \sqrt{1 + \frac{1}{\omega^2}}$$

$$= \frac{-1 \pm \sqrt{1 + \frac{1}{\omega^2}}}{\omega}$$

$$= \frac{-1 \pm \sqrt{1 + \frac{1}{\omega^2}}}{\omega}$$