

$$f(f(a)) = f\left(\frac{a+1}{a-1}\right) \rightarrow \frac{a+1}{a-1} - 1 = \frac{a+1-(a-1)}{a-1} = \frac{a+1-a+1}{a-1} = \frac{2}{a-1}$$

$$\rightarrow \frac{2a}{2} = (a) \checkmark \rightarrow \boxed{a \in \mathbb{R} - \{1\}}$$

$$g(a) = \sqrt{a^2 + 1}$$

$$f(g(a)) = f(\sqrt{a^2 + 1}) \rightarrow (\sqrt{a^2 + 1})^2 - \varepsilon(\sqrt{a^2 + 1})$$

$$(\sqrt{a^2 + 1})^2 = a^2 + 1 \rightarrow \boxed{f(g(a)) = a^2 + 1 - \varepsilon\sqrt{a^2 + 1}}$$

$$x + y = \varepsilon \rightarrow \underline{x = 1}$$

$$f(\varepsilon) = (1)^2 = 2(1) + 4 \rightarrow \boxed{f(\varepsilon) = 2}$$

$$x + y = -y \rightarrow \underline{x = -y} \rightarrow f(-y) = (-y)^2 = 2(-y) + 4 \rightarrow \boxed{f(-y) = 4 - 2y}$$

$$f(\varepsilon)(f(-y)) = 2 \times (4 - 2y) = 8 - 4y = 1 \varepsilon \varepsilon \rightarrow \boxed{8 - 4y} \checkmark$$

$$\boxed{\cos x = t}$$

$$t^2 + \sin^2 x = 1 \rightarrow t^2 + \sin^2 x - 1 = 0$$

$$1t^2 + 1\sin^2 x - 1 = 0 \rightarrow x = \frac{-1 \pm \sqrt{1}}{1} \begin{cases} t_1 = 1 \checkmark \\ t_2 = -1 \checkmark \end{cases}$$

الف) $x \neq 0$ و $x-1 \neq 0 \rightarrow x \neq 1$ $\left\{ \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \xrightarrow{\text{مخرج مشترک}} \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{-2x+1}{x(x-1)} \geq 0 \right.$
 $D_f = \{0, \frac{1}{2}\} \cup (1, +\infty)$ $\leftarrow \text{نقش علامت}$

$\rightarrow x \neq -1$ و $x \neq 0$ و $x \neq 1$ و $x \neq -2$ $\left\{ \frac{x}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow D_f = \mathbb{R} - \left\{ -2, -\frac{5}{2} \right\} \right.$

الف) $\left(\frac{1}{x} - 4 \right) (3x - 5) \geq 0 \xrightarrow{\text{نقش علامت}} D_f = (-\infty, -2] \cup [2, 5, +\infty)$
 $\boxed{x = -2}$ $\boxed{x = \frac{5}{3}}$

$\rightarrow x-1 \geq 0 \rightarrow x \geq 1$ و $y+1 \geq 0 \rightarrow y \geq -1 \rightarrow D_f = \{(x, y) \in \mathbb{R}^2 \mid x \geq 1, y \geq -1\}$

$(x^2 - x - 2) \geq 0 \rightarrow x = -1, 2 \rightarrow \text{نقش علامت} \Rightarrow (-\infty, -1) \cup (2, +\infty)$

$x^2 - 1 \geq 0 \rightarrow x \geq 1 \rightarrow (-\infty, -1) \cup (1, +\infty)$

$\xrightarrow{\text{نقش علامت}} D_f = (-\infty, -1) \cup (2, +\infty)$

$3 + ax - x^2 \geq 0 \rightarrow -x^2 + ax + 3 \geq 0 \rightarrow x_1 = -2$
 $\xrightarrow{x=2} -2 - 2a + 3 = 0 \rightarrow -1 - 2a = 0 \rightarrow \boxed{a = -\frac{1}{2}}$
 $-x^2 - \frac{1}{2}x + 3 = 0 \xrightarrow{\times 2} 2x^2 + x - 6 = 0 \rightarrow \boxed{b = 1, 5}$ $\left. \begin{matrix} \boxed{a = -\frac{1}{2}} \\ \boxed{b = 1, 5} \end{matrix} \right\} \boxed{a+b=1}$

$f(x) - x \geq 0 \rightarrow f(x) \geq x$

① $x \geq 1 \rightarrow 3x - 2 \geq x \rightarrow x \geq 1$
 ② $x < 1 \rightarrow 2x + 3 \geq x \rightarrow x \geq -3$ $\left\{ \begin{matrix} \text{نقش علامت} \\ \Rightarrow -3 \leq x < 1 \end{matrix} \right.$

$D_g = [-3, +\infty)$

$\sin \frac{\mu}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}} \text{ , } \cos 2\mu = 1$

$\frac{\sqrt{3}}{\sqrt{3}} + 1 \rightarrow \frac{\sqrt{3} + 1}{\sqrt{3}}$