

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$ $\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} = \frac{-2x+1}{x^2-x} \geq 0$ $x=1$
 $x=0$
 $x=1$
 $D_f = (0, \frac{1}{2}] \cup (1, +\infty)$

ب) $f(x) = \frac{1}{x+1} - \frac{1}{x} = \frac{x-1-x}{x^2+x} = \frac{-x-1}{x^2+x}$ $I: x^2+x \neq 0 \rightarrow x(x+1) \neq 0$
 $D_{f(x)} = \mathbb{R} - \{-\frac{1}{2}, -1, 0, 1\}$ $II: \frac{f(x+a)}{x^2+x-1} \neq 0 \rightarrow f(x+a) \neq 0$
 $x=1$ $x=-1$ $x=0$

الف) $F(x) = \sqrt{\left[\left(\frac{1}{x}\right)^x - 9\right] \left[1 - \frac{1}{x}\right] \left[\left(\frac{1}{x}\right)^x - 9\right] \left[1 - \frac{1}{x}\right]} \geq 0 \rightarrow x \mid \begin{matrix} -1 & 0 \\ + & - & + \end{matrix}$
 $\frac{1}{x} = 9 \rightarrow x = \frac{1}{9}$ $x = -1$ $x = 0$ $D_f = (-\infty, -1] \cup [\frac{1}{9}, +\infty)$
 ب) $\sqrt{x-1} + \sqrt{9+1} = 4$

$F(x) = \frac{\log x}{\sqrt{x^2-1} + 1}$ $I: x^2-1 > 0 \rightarrow x < -1 \text{ or } x > 1$ $x \mid \begin{matrix} -1 & +1 \\ + & - & + \end{matrix}$
 $II: \sqrt{x^2-1} + 1 \neq 0 \rightarrow \sqrt{x^2-1} \neq -1 \rightarrow \mathbb{R}$
 $III: x^2 - x - 2 > 0 \rightarrow x \mid \begin{matrix} -1 & 2 \\ + & - & + \end{matrix} \rightarrow (-\infty, -1] \cup [2, +\infty)$
 $D_f = (-\infty, -1] \cup [2, +\infty) \cap \mathbb{R} \cap (-\infty, -1] \cup [2, +\infty) = (-\infty, -1] \cup [2, +\infty)$

$F(x) = \sqrt{1+a-x^2} \rightarrow F(x) = \sqrt{1-\frac{1}{x}-x^2}$ $a+b = -\frac{1}{x} + \frac{1}{x} = \frac{1}{x} = 1$
 $x=1 \rightarrow 1 + a - 1 = 0 \rightarrow a = 0$
 $x=-1 \rightarrow 1 - a - 1 = 0 \rightarrow a = 0$
 $x=0 \rightarrow 1 + a = 0 \rightarrow a = -1$
 $a+b = \frac{c}{a} \rightarrow -1 \times b = \frac{1}{-1} \rightarrow b = \frac{1}{-1}$

$g(x) = \sqrt{F(x)-x} \rightarrow x > 1 \quad g(x) = \sqrt{1-x-x} \rightarrow g(x) = \sqrt{1-2x} \quad x > 1$
 $\rightarrow x < 1 \quad g(x) = \sqrt{1+x-x} \rightarrow g(x) = \sqrt{1} \quad x < 1$

$$F(x) = r x^r - r x$$

$$F\left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}\right) = F\left(\frac{\sqrt{r}}{r} + 1\right) = r\left(\frac{\sqrt{r}}{r} + 1\right)^r - r\left(\frac{\sqrt{r}}{r} + 1\right) = r\left(\frac{r}{r} + \sqrt{r} + 1\right) - r\sqrt{r} - r$$

$$\frac{r}{r} + r\sqrt{r} + r - r\sqrt{r} - r = \frac{r}{r} - r = -\frac{1}{r}$$

$$F(x) = \frac{x+1}{x-1} \quad F(F(a)) = a \rightarrow F(F(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{r_a}{r} = a$$

$$a \in \mathbb{R} - \{1\}$$

$$f(x) = x^r - r x \quad f(x) = -\varepsilon \rightarrow x^r - r x = -\varepsilon \rightarrow x^r - \varepsilon x + \varepsilon = 0 \rightarrow (x-r)^r = 0 \rightarrow x=r$$

$$g(x) = \sqrt{x^r - 1} \quad g(a) = r \rightarrow \sqrt{x^r - 1} = r \xrightarrow{\text{square}} x^r - 1 = r^2 \rightarrow x^r = r^2 + 1 \rightarrow x = \pm \sqrt{r^2 + 1}$$

$$F(g(a)) = -\varepsilon \quad a = \pm \sqrt{r^2 + 1}$$

$$F(x+r) = x^r - r x + r$$

$$f(r) \times f(r) = r \times r = r^2$$

$$x+r = \varepsilon \rightarrow x = \varepsilon - r$$

$$f(r) = 1 - r + r = 1$$

$$x+r = -r \rightarrow x = -2r$$

$$f(-r) = (-r)^r - r(-r) = r^r + r$$

$$f(\cos x + \omega \cos x) = \tan^r x$$

$$r \cos^r x + \omega \cos = \frac{r \omega}{10} \rightarrow r \cos^r x + \omega \cos - \frac{r \omega}{10} = 0 \rightarrow \Delta = b^2 - 4ac \rightarrow r^2 - (r^2 \cos^2 x - \frac{r \omega}{10}) = 9 \rightarrow \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow \frac{-\omega \pm r}{2}$$

$$\cos x = -1 \rightarrow f(r \omega) = \cos^r x = -1$$

$$\cos x = -\frac{1}{r} \rightarrow f(r \omega) = \tan^r x \rightarrow \tan^r x = \frac{1}{\cos^r x} \rightarrow \tan^r x = \frac{1}{(-\frac{1}{r})^r} = -1 \rightarrow \tan^r x = 1$$