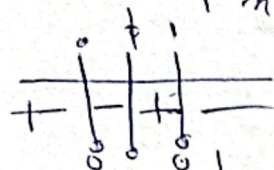


۲. افین عالی نوری

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} = \sqrt{\frac{x^2 - 2x + 1 - x^2}{x(x-1)}} = \sqrt{\frac{-2x+1}{x(x-1)}}$



$\Rightarrow Df = (-\infty, 0) \cup [\frac{1}{2}, \infty)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{1}{x}}{\frac{1}{x-1} + \frac{1}{x+2}} \Rightarrow \frac{x+1 \neq 0 \Rightarrow x \neq -1}{x \neq 0 \Rightarrow x \neq 0}, \frac{x-1 \neq 0 \Rightarrow x \neq 1}{x+2 \neq 0 \Rightarrow x \neq -2}$
 $\frac{1}{x+1} - \frac{1}{x} = \frac{x - (x+1)}{x(x+1)} = \frac{-1}{x(x+1)}$
 $\frac{1}{x-1} + \frac{1}{x+2} = \frac{x+2 + x-1}{(x-1)(x+2)} = \frac{2x+1}{(x-1)(x+2)}$
 $\frac{-1}{x(x+1)} \cdot \frac{(x-1)(x+2)}{2x+1} \neq 0 \Rightarrow \frac{-(x-1)(x+2)}{x(x+1)(2x+1)} \neq 0$
 $\Rightarrow Df = \mathbb{R} - \{-2, -\frac{1}{2}, -1, 0, 1\}$

الف) $f(x) = \sqrt{(\frac{1}{x})^2 - 9} \sqrt{32 - 2^x} \Rightarrow ((\frac{1}{x})^2 - 9) (32 - 2^x) \geq 0$
 $\Rightarrow Df = (-\infty, -2] \cup [2, \infty)$
 $(\frac{1}{x})^2 = 9 \Rightarrow x = \pm \frac{1}{3}$
 $32 - 2^x = 0 \Rightarrow 2^x = 32 \Rightarrow x = 5$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{y+1} = 3 - \sqrt{x-1} \Rightarrow \textcircled{1} x-1 \geq 0 \Rightarrow x \geq 1$
 $\textcircled{2} 3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x-1 \leq 9 \Rightarrow x \leq 10$
 $Df = [1, 10]$

$f(x) = \log_x (x^2 - x - 2)$

$\textcircled{1} x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0$



برای معادلات منفرجه و از آنجا که
 معادله منفرجه است و از آنجا که
 باید معادله منفرجه باشد و از آنجا که
 باید معادله منفرجه باشد و از آنجا که

$\textcircled{2} x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow x \geq 1 \wedge x \leq -1$

$\textcircled{1} \wedge \textcircled{2} \Rightarrow Df = (-\infty, -1) \cup (2, \infty)$

$f(x) = \sqrt{-x^2 + 4x + 3} \Rightarrow Df = [-2, 6]$

$x = -2 \Rightarrow -x^2 + 4x + 3 = 0 \Rightarrow -4 + 4x + 3 = 0 \Rightarrow 4x - 1 = 0 \Rightarrow x = \frac{1}{4}$

$f(x) = \sqrt{-x^2 - 4x + 6} \Rightarrow \frac{c}{a} = -2b = -3 \Rightarrow b = \frac{3}{2}$

$\Rightarrow a+b = \frac{3}{2} - \frac{1}{2} = 1$

$f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases}$

$g(x) = \sqrt{f(x) - x} \Rightarrow \begin{cases} x \geq 1 : \sqrt{3x-2-x} = \sqrt{2x-2} = g(x) \Rightarrow 2x-2 \geq 0 \Rightarrow x \geq 1 \\ x < 1 : g(x) = \sqrt{2x+3-x} = \sqrt{x+3} \Rightarrow x+3 \geq 0 \Rightarrow x \geq -3 \end{cases}$

$\Rightarrow Dg = [-3, +\infty)$

$$f(x) = x^2 - \varepsilon x$$

$$f\left(\sin \frac{\pi}{4} + i \cos \frac{\pi}{4}\right) = f\left(\frac{\sqrt{2}}{2} + i\right) = \left(\frac{\sqrt{2}}{2} + i\right)^2 - \varepsilon \left(\frac{\sqrt{2}}{2} + i\right) = \left(\frac{1}{2} - \frac{1}{2} + i\sqrt{2}\right) - \varepsilon \left(\frac{\sqrt{2}}{2} + i\right) = i\sqrt{2} - \varepsilon \frac{\sqrt{2}}{2} - \varepsilon i = i\sqrt{2} - \varepsilon \frac{\sqrt{2}}{2} - \varepsilon i$$

6

$$f(x) = \frac{x+1}{x-1} \quad f(f(a)) = a$$

$$f(a) = \frac{a+1}{a-1} \Rightarrow f\left(\frac{a+1}{a-1}\right) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{\frac{2a}{a-1}}{\frac{2}{a-1}} = a$$

$$\frac{a(a-1)}{(a-1)} = a \Rightarrow \text{با ازنمایی} \Rightarrow \text{با ازنمایی} \Rightarrow a=1 \text{ جزو } \mathbb{R} \text{ نیست} \Rightarrow a \in \mathbb{R} - \{1\}$$

7

$$f(x) = x^2 - \varepsilon x \quad g(x) = \sqrt{x^2 - 1} \quad f(g(a)) = -\varepsilon$$

$$f(g(a)) = -\varepsilon \Rightarrow a^2 - \varepsilon a = -\varepsilon \Rightarrow a^2 - \varepsilon a + \varepsilon = 0 \Rightarrow (a-1)^2 = 0 \Rightarrow a = 1$$

$$\Rightarrow \frac{1}{2} \pi = 1 \Rightarrow g(a) = 1$$

$$\sqrt{a^2 - 1} = 1 \Rightarrow a^2 - 1 = 1 \Rightarrow a^2 = 2 \Rightarrow a = \pm \sqrt{2}$$

8

$$f(n+c) = n^2 - \varepsilon n + c = (n-1)(n-1) \Rightarrow (n+c-1)(n+c-1)$$

$$\Rightarrow f(a) = (a-1)(a-1)$$

$$\Rightarrow f(1) = (-1)(-1) = 1, \quad f(-1) = (-1)(-1) = 1$$

$$\Rightarrow f(1) \times f(-1) = 1 \times 1 = 1$$

9

$$f(\varepsilon \cos^2 x + \delta \cos x) = \tan^2 x$$

$$\Rightarrow \varepsilon \cos^2 x + \delta \cos x - \frac{1}{\varepsilon} = 0$$

$$\Rightarrow \left(\varepsilon \cos x + \frac{1}{\varepsilon}\right)(\varepsilon \cos x - 1) = 0 \Rightarrow \varepsilon \cos x = \frac{1}{\varepsilon} \Rightarrow \cos x = \frac{1}{\varepsilon^2}$$

$$\cos x = \frac{1}{\varepsilon^2}, \quad \text{پس } 1 + \tan^2 x = \frac{1}{\cos^2 x} \Rightarrow 1 + \tan^2 x = \varepsilon^2 \Rightarrow \tan^2 x = \varepsilon^2 - 1$$

$$\Rightarrow |f(\varepsilon, \delta)| = 3$$

10

♥ بانم یونس ♥ قرینیت ♥ خیر و خوبی هاست

(من طلبه‌ام پس شما ایوب‌های با نور طلبم)