

$$f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \xrightarrow{L'H} \frac{(x-1)^{\frac{1}{2}} - x^{\frac{1}{2}}}{x(x-1)} \xrightarrow{\frac{x^{\frac{1}{2}} - \cancel{1} - x^{\frac{1}{2}}}{x(x-1)}} \frac{1 - 2x}{x(x-1)} \xrightarrow{c, 1} \frac{1}{c}$$

$\frac{1}{x} \rightarrow (-\infty, 0) \cup [\frac{1}{2}, +\infty)$

$$f(x) = \frac{1}{x+1} - \frac{1}{x} \sim \frac{x-1x-1}{x(x+1)} \rightarrow \frac{-x-1}{x(x+1)} \rightarrow \frac{-1}{x} \Rightarrow \{ -1, -\frac{1}{x}, 1, 0, 1 \}$$

$$f(x) = \sqrt{\left(\left(\frac{1}{r}\right)^x - 9\right)(r^x - r^x)} \rightarrow \frac{-r \log_e r}{+ \cancel{0} - \cancel{0} +} \quad (-\infty, -r] \cup [\log_e r, +\infty) \text{ (i)}$$

$\sqrt{x-1} + \sqrt{y+1} = 3 \rightarrow \sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow y+1 = \sqrt{x-1} - 9\sqrt{x-1} + 9$
 $\rightarrow \sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow$ باید هر دو را مثبت باشد $\rightarrow \sqrt{x-1} \leq 3 \rightarrow x-1 \leq 9 \rightarrow x \leq 10$
 $y^2 \sqrt{x-1} - 9\sqrt{x-1} + 9 \rightarrow \sqrt{x-1} = 0 \rightarrow x=1 \rightarrow (1, +\infty) \rightarrow \phi$
 $D_f \subset [0, 10]$

$\log_5 x^2 - x - 2 \rightarrow x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow \frac{-1}{+} \frac{2}{-}$
 $\frac{\sqrt{x^2-1} + 1}{=0}$
 $\sqrt{x^2-1} > 0 \rightarrow (x-1)(x+1) > 0 \rightarrow \frac{-1}{+} \frac{1}{-} \rightarrow (-\infty, -1] \cup [1, +\infty)$

$$p(x) = \sqrt{-x^2 + ax + r} \xrightarrow{\text{ans b}} [-r, b] \xrightarrow{x=r} -r - ra + r = 0 \rightarrow -ra = 0 \rightarrow a = \frac{-1}{r}$$

$$-x^2 - \frac{1}{r}x + r = 0 \rightarrow \frac{1}{r} \pm \frac{\sqrt{1+4r^2}}{2r} = r - r = 0 \quad \left(\frac{r}{r} = b \right) \quad a + b = \frac{1}{r} + \frac{r}{r} = 1$$

$$f(x) = \begin{cases} x-1 & x \geq 1 \\ x+1 & x < 1 \end{cases} \quad \left\{ (-\infty, -1] \cup [1, +\infty) \right\} \leftarrow \frac{-1}{+ \quad - \quad +}$$

$$g(x) = \sqrt{f(x) - x} \geq 0 \xrightarrow{c} \begin{cases} x-1 = x \rightarrow x-1=0 \rightarrow x=1 \\ x+1 = x \rightarrow x+1=0 \rightarrow x=-1 \end{cases}$$

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 $\rightarrow Df = [-1, +\infty)$
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چون ۲ ضابطہ ای است نہ کوئی یقین علامت را با جدول انجم بدھی!

$f(x) = 2x^2 - 5x \rightarrow 2x(x - \frac{5}{2})$ $f(\sin \frac{\pi}{4} + \cos 2\pi) \rightarrow f(\frac{\sqrt{2}}{2} + 1) = 2(\frac{\sqrt{2}}{2} + 1)(\frac{\sqrt{2}}{2} + 1 - \frac{5}{2})$ $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\cos 2\pi = 1$ $2(\frac{\sqrt{2}}{2} - 1) = -(\frac{2 - \sqrt{2}}{1})$ نزدج	6
$f(x) = \frac{x+1}{x-1} \quad f(f(a)) = a$ $f(a) = \frac{a+1}{a-1} \rightarrow f(\frac{a+1}{a-1}) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{a+1+a-1}{a-1}}{\frac{a+1-a+1}{a-1}} = \frac{2a}{2} = a$ $a \neq 1$ $\mathbb{R} - \{1\}$ جواب	7
$f(x) = x^2 - 5x \quad f(g(a)) = -8$ $g(x) = \sqrt{x^2 - 1} \quad a = ?$ $f(?) = -8 \rightarrow x^2 - 5x = -8 \rightarrow x^2 - 5x + 8 = 0 \rightarrow (x-2)(x-3) = 0 \rightarrow x = 2$ $g(a) = 2 \rightarrow \sqrt{a^2 - 1} = 2 \rightarrow a^2 - 1 = 4 \rightarrow a^2 = 5 \rightarrow a = \pm \sqrt{5}$	8
$f(x+3) = x^2 - 5x + 9$ $f(2) \xrightarrow{x=1} 1 - 5 + 9 = 5$ $f(-3) \xrightarrow{x=-4} 16 - 20 + 9 = 5$ $f(2) \times f(-3) = 5 \times 5 = 25$	9
$f(\cos^2 x + \sin^2 x) = \tan^2 x$ $f(1) \rightarrow \cos^2 x + \sin^2 x = 1 \rightarrow \cos^2 x + \sin^2 x = 1 \rightarrow \cos^2 x = 1 \rightarrow \cos x = \pm 1$ $1 = 2\cos^2 x + \sin^2 x = 1 \rightarrow \frac{-1 \pm \sqrt{1}}{2} \rightarrow \frac{-1 \pm 1}{2} \rightarrow \frac{0}{2} = 0$ جواب $\tan^2 \theta = (\sqrt{3})^2 = 3$ جواب	10

الف ۱) $\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{-x+1}{x(x-1)} \geq 0$

$\frac{1}{x} - \frac{1}{x-1} \geq 0 \rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

Sign chart: $\frac{1}{x} - \frac{1}{x-1}$ has sign changes at $x=0$ and $x=\frac{1}{2}$. The sign is positive for $x < 0$ and $0 < x < \frac{1}{2}$, and negative for $x > \frac{1}{2}$.

ب ۲) $\sqrt{x-1} + \sqrt{y+1} = 3$, $\sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow \sqrt{x-1} \geq 0 \rightarrow \sqrt{x-1} \leq 3 \rightarrow x \leq 10$

$x-1 \geq 0 \rightarrow x \geq 1$ I

I, II $\rightarrow 1 \leq x \leq 10 \rightarrow D_f = [1, 10]$

$f(x) - x = \begin{cases} x^2 - 2 - x & x \geq 1 \\ x^2 + 3 - x & x < 1 \end{cases} = \begin{cases} x^2 - 2 & x \geq 1 \\ x + 3 & x < 1 \end{cases}$

$f(x) - x \geq 0 \rightarrow \begin{cases} x^2 - 2 \geq 0 \rightarrow x \geq \sqrt{2} \checkmark \\ x + 3 \geq 0 \rightarrow x \geq -3 \checkmark \end{cases}$

$\rightarrow D_f = [-3, +\infty)$

باید به دقت نیز مشخص است!

