

سوال 5
پایان روز 13

$$f(x), \frac{(x-1)^2 - x^2}{x(x-1)} = \frac{x^2 + 1 - 2x - x^2}{x(x-1)} = \frac{1-2x}{x(x-1)} \quad \begin{matrix} + & - & + \\ - & + & - \end{matrix} \quad D_f: (-\infty, 0) \cup [\frac{1}{2}, 1) \quad -1$$

$$f(x), \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}} \Rightarrow x \neq 0, x \neq -1, x \neq 1, x \neq -2$$

$$\frac{3}{x-1} + \frac{1}{x+2} \neq 0 \Rightarrow \frac{3x+4+x-1}{x^2+x-2} \neq 0 \Rightarrow 2x+3 \neq 0 \Rightarrow x \neq -\frac{3}{2} \quad D_f: \mathbb{R} - \{-\frac{3}{2}, -1, 0, 1\}$$

$$f(x) = \sqrt{((\frac{1}{x})^2 - 4)(3x - x^2)} \quad ((\frac{1}{x})^2 - 4)(3x - x^2) \geq 0 \quad \begin{matrix} - & + \\ + & - \end{matrix} \quad D_f: (-\infty, -2] \cup [1, +\infty)$$

$$(b) \sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{y+1} = 3 - \sqrt{x-1} \quad \begin{matrix} x-1 \geq 0 \Rightarrow x \geq 1 \\ 3 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 3 \Rightarrow x \leq 10 \end{matrix} \quad D_f: [1, 10]$$

$$f(x), \frac{\log \frac{(x^2-x-2)}{x^2-1}}{\sqrt{x^2-1}+1} \quad \begin{matrix} x^2-x-2 > 0 \Rightarrow (x-2)(x+1) > 0 \\ x^2-1 > 0 \Rightarrow x > 1, x < -1 \end{matrix} \quad \begin{matrix} - & + \\ + & - \end{matrix} \quad D_f: (-\infty, -1) \cup (2, +\infty)$$

$$f(x) = \sqrt{3+ax-x^2} \quad D_f: [-2, b] \quad \begin{matrix} x=-2 \Rightarrow 3-2a-4=0 \Rightarrow a=-\frac{1}{2} \\ x=b \Rightarrow 3+ab-b^2=0 \Rightarrow b=\frac{3}{2} \end{matrix} \quad D_f: [-2, \frac{3}{2}]$$

$$g(x) = \sqrt{f(x)} - x \quad \begin{cases} x \geq 1 \Rightarrow \sqrt{3x-2-x} = \sqrt{2x-2} \Rightarrow 2x-2 \geq 0 \Rightarrow x \geq 1 \\ x < 1 \Rightarrow \sqrt{3x+3-x} = \sqrt{2x+3} \Rightarrow 2x+3 \geq 0 \Rightarrow x \geq -\frac{3}{2} \end{cases} \quad D_g: [-\frac{3}{2}, +\infty)$$

$$f(x), 2x^2 - 2x \quad f(\sin \frac{\pi}{6} + \cos \frac{\pi}{6}), f(\frac{\sqrt{3}}{2} + 1) = 2(\frac{\sqrt{3}}{2} + 1)^2 - 2(\frac{\sqrt{3}}{2} + 1) = 2(\frac{3}{4} + 1 + \sqrt{3}) - 2(\frac{\sqrt{3}}{2} + 1) = \frac{3}{2} + 2 + 2\sqrt{3} - \sqrt{3} - 2 = \frac{3}{2} + \sqrt{3}$$

$$f(f(a)) = a \quad f(x), \frac{x+1}{x-1} \Rightarrow f(a), \frac{a+1}{a-1} \Rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1+a-1}{a-1} = \frac{2a}{a-1} \Rightarrow -1$$

برای اینکه منجر به تناقض نشود
 $a \neq 1$

$$f(x), x^2 - 2x \quad g(x) = \sqrt{x^2 - 1} \Rightarrow g(x), \sqrt{x^2 - 1} \Rightarrow f(g(x)) = (\sqrt{x^2 - 1})^2 - 2\sqrt{x^2 - 1} \Rightarrow -1$$

$$x^2 - 1 - 2\sqrt{x^2 - 1} = -2 \Rightarrow x^2 + 1 = 2\sqrt{x^2 - 1} \Rightarrow x^2 - 1 = \frac{x^2 + 1 - 2}{1} = \frac{x^2 - 1}{1}$$

$$14x^2 - 14 = x^2 + 1 + 9x^2 \Rightarrow x^2 - 1 = x^2 + 2 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$$

$$1 \pm \sqrt{1 - 1} = 2 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$$

$$f(x+3), x^2 - 2x + 4 \Rightarrow f(2), f(1+3), 1 - 2 + 4 = 3 \Rightarrow f(-3), f(-4+3) = 14 + 3 + 4 = 21$$

$$f(2), f(-3), 21 \times 2, 122$$

$$f(\cos x + 2 \cos x), \tan x \Rightarrow 2 \cos x + 2 \cos x = 4 \cos x \Rightarrow 2 \cos x + 2 \cos x = 4 \cos x$$

$$(2 \cos x + 2 \cos x)(2 \cos x - 1) = 0 \Rightarrow \cos x = \frac{1}{2} \sqrt{1} \quad \cos x = \frac{1}{2} \times$$

$$\cos x = \frac{1}{2} \quad 1 + \tan x = \frac{1}{\cos x} \Rightarrow 1 + \tan x = 2 \Rightarrow \tan x = 1$$

$$\tan(1, 2), 3$$