

۱) الف) $\sqrt{\frac{x-1}{x}} - \frac{x}{x-1}$
 $\frac{1-x^2}{x(x-1)} \geq 0$ $x_1=0, x_2=\frac{1}{x}$
 $D_f: (-\infty, 0) \cup (\frac{1}{x}, 1)$
 ب) $x \neq -2, -1, 0, 1$
 $\frac{x}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{exed}{(x-1)(x+2)} \neq 0 \rightarrow x \neq -\frac{2}{x}$
 $D_f: \mathbb{R} - \{-2, -\frac{2}{x}, -1, 0, 1\}$

الف) $\sqrt{((\frac{1}{x})^2 - 9)(x^2 - 2x)}$
 $(\frac{1}{x^2} - 9)(x^2 - 2x) \geq 0$
 $(\frac{1}{x^2} - 9)(x^2 - 2x) \geq 0$
 ب) $x-1 \geq 0 \rightarrow x \geq 1$
 $\sqrt{x-1} \leq 2 \rightarrow x-1 \leq 4 \rightarrow x \leq 5$
 $D_f: [1, 5]$

$x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow x_1=-1, x_2=2 \rightarrow x \in (-\infty, -1) \cup (2, +\infty)$
 $x^2 - 1 \geq 0 \rightarrow x \in (-\infty, -1] \cup [1, +\infty)$
 $\sqrt{x^2-1} \geq 1 \rightarrow \sqrt{x^2-1} \geq 1 \rightarrow x^2-1 \geq 1 \rightarrow x^2 \geq 2 \rightarrow x \leq -\sqrt{2} \text{ or } x \geq \sqrt{2}$
 $D_f: (-\infty, -1) \cup (2, +\infty)$

$3 \leq ax - x^2 \leq 0$ $x_1=-2, x_2=b$
 $(-2)^2 \leq a(-2) - 3 \leq 0 \rightarrow 4 \leq -2a - 3 \leq 0 \rightarrow -2a \geq 7 \rightarrow a \leq -\frac{7}{2}$
 $3, x_1, x_2 \rightarrow -\frac{b}{a}$
 $-2 \leq b \rightarrow -\frac{1}{a} \leq b \rightarrow b \geq -\frac{1}{a}$
 $D_f: (-\infty, -1) \cup (2, +\infty)$

الف) $f(x) - x \geq 0 \rightarrow f(x) \geq x$
 $x \geq 1$
 $x^2 - 2 \geq x \rightarrow x^2 \geq x+2 \rightarrow x \geq 1$
 $x < 1$
 $x^2 + 3 \geq x \rightarrow x^2 \geq x-3 \rightarrow x \in [-3, 1]$
 $D_f: [-3, 1]$

$\sin \frac{\pi}{r}, \sqrt{\frac{r}{r}}$ $\cos \frac{\pi}{r} = 1$ $\sqrt{\frac{r}{r}} \leq 1 \Rightarrow \frac{\sqrt{r-r}}{r}$ $r(\frac{\sqrt{r-r}}{r}) = \sqrt{r-r}$ $(x-r) \rightarrow \frac{\sqrt{r-r}}{r} - r, \frac{\sqrt{r-r}}{r}$ $f(x) = rx^2 - \varepsilon x$ $rx(x-r)$ $(\sqrt{r-r}) \times (\frac{\sqrt{r-r}}{r})$ $(\sqrt{r-r})(\sqrt{r-r}) \rightarrow \frac{r-r}{r}$	6
$f(a) = \frac{a-1}{a-1}$ $f(f(a)) \rightarrow \frac{f(a)-1}{f(a)-1} \rightarrow \frac{\frac{a-1}{a-1}-1}{\frac{a-1}{a-1}-1} \rightarrow \frac{\frac{a-1-a+1}{a-1}}{\frac{a-1-a+1}{a-1}} \rightarrow \frac{\frac{-1}{a-1}}{\frac{-1}{a-1}} \rightarrow 1$ $a-1 \neq 0$ $a \neq 1$ $f(a) \neq 1 \Rightarrow \frac{a-1}{a-1} \neq 1 \rightarrow a-1 \neq a-1 \rightarrow 1 \neq 1 \checkmark$	7
$f(g(a)) \rightarrow (g(a))^r - \varepsilon g(a) = -\varepsilon$ $(g(a))^r - \varepsilon g(a) + \varepsilon = 0$ $(g(a)-r)^r = 0$ $g(a)-r = 0 \rightarrow g(a) = r$ $\sqrt{a^r-1} = r \Rightarrow a^r = 1 + \varepsilon$ $a^r, \delta \rightarrow a, \pm \sqrt{\delta}$ $a = \sqrt{\delta}$ $a = -\sqrt{\delta}$	8
$f(-r)$ $x \in \mathbb{R}, -r \rightarrow x, -r$ $f(x)$ $x \in \mathbb{R}, \varepsilon$ $x = 1$ $f(x) = 1$ $\forall x \in \mathbb{R}, \varepsilon$	9
$(\varepsilon \cos^2 x + \delta \cos x, r, \delta, \frac{1}{r}) \leq r$ $\wedge \cos^2 x \in [0, 1] \Rightarrow \cos x = 1 \Rightarrow \Delta = \sqrt{r^2 \varepsilon} = 1$ $(\cos^2 x + \delta \cos x) \leq r$ $\tan^2 x = \frac{1}{\cos^2 x} - 1 \rightarrow \frac{1}{\cos^2 x} = 1 + \frac{1}{r}$ $\frac{1}{\varepsilon} = 1, \varepsilon = 1, \frac{1}{r}$ $\frac{1}{\varepsilon}$	10